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The influence of roughness structure on flow resistance on steep slopes

L'influence de la structure de rugosité sur la résistance à des écoulements en fortes pentes

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ABSTRACT

There is no standard flow resistance equation for the determination of mean flow velocity in mountain streams. The reason lies in the morphology of mountain streams, i.e. steep slopes, large roughness elements, bed forms and water depths of the same order of magnitude as the bed material size. Logarithmic, Froude and power-law approaches to determine flow resistance are discussed with respect to the roughness parameter which is usually a characteristic grain size. As a result of the irregular nature of gravel-bed profiles it is shown that the structure of these stream beds cannot be described sufficiently by a characteristic percentile of the grain size distribution. Statistical properties of a series of bed profiles are investigated in order to quantify the effect of roughness on flow resistance. The standard deviation of the bed elevations is introduced as characteristic roughness length and its applicability is verified by the analysis of experimental data. Based on this roughness parameter, an approach for the determination of flow resistance is derived which allows for spatial averaging of the flow field. Conclusions concerning the influence of bed forms on flow resistance are drawn with the use of the concept of the "at-a-site" hydraulic geometry.

RÉSUMÉ

Il n'y a aucune équation standard de résistance à l'écoulement pour la détermination de la vitesse moyenne dans des torrents de montagne. La raison se situe dans la morphologie des torrents de montagne, i.e. des pentes fortes, des éléments de rugosité de grande taille, des formes de lit et des profondeurs d'eau du même ordre de grandeur que la taille du matériau de lit. Les approches par des lois logarithmique, de Froude et en puissance pour déterminer la résistance à l'écoulement sont discutées en fonction du paramètre de rugosité qui est habituellement une taille de grain caractéristique. En raison de la nature irrégulière des profils du gravier de lit, on montre que la structure de ces lits de torrents ne peut pas être décrite correctement par un centile caractéristique de la distribution granulométrique. Des propriétés statistiques d'une série de profils de lit sont étudiées afin de mesurer l'effet de la rugosité sur la résistance à l'écoulement. L'écart type des hauteurs de lit est introduit comme dimension caractéristique de la rugosité et son applicabilité est vérifiée par l'analyse des données expérimentales. A partir de ce paramètre de rugosité, on déduit une approche pour déterminer la résistance à l'écoulement ce qui permet de considérer une moyenne spatiale de cet écoulement. Concernant l'influence des formes de lit sur la résistance à l'écoulement, des conclusions sont tirées introduisant le concept de géométrie hydraulique << sur site >>.

Keywords: Friction factor; steep slopes; hydraulic geometry; roughness.

1 Introduction

In spite of the wealth of literature on the hydraulics of streams and channels, the hydraulics of steep mountain streams is poorly understood relative to that of lowland streams. So far, there is no standard flow resistance equation for the determination of mean flow velocity in mountain streams to be found in the literature. The reason lies in the morphology of mountain streams which is characterised by steep slopes, large roughness elements and water depths of the same order of magnitude as the bed material size. Additionally, in streams with gradients larger than 3%, the riverbed is often characterised by a staircase like morphology, commonly called step-pool systems. General classifications of bed forms in mountain streams are available [19,40]. Step-pool systems may develop as a result of selective erosion [38] and they lead to a wavy and undulating water surface with non-uniform flow conditions as shown by Fig. 1.

Most approaches for the determination of flow resistance in mountain streams use the general concept of bed shear stress. This concept is based on uniform flow conditions and was originally developed for flows with gentle slopes. As these uniform conditions do not apply to mountain streams, approaches based on this concept have to be considered as empirical procedures. A further reason for the empiricism of present approaches results from data acquisition techniques used in mountain streams. Due to the complex step-pool morphology and roughness elements which are large relative to water depth (see Fig. 1), it is not possible to define an obvious bed-level datum and the definition of water depth is rather arbitrary in the literature [41]. Furthermore, hydraulic radius based on wetted perimeter has a fractal

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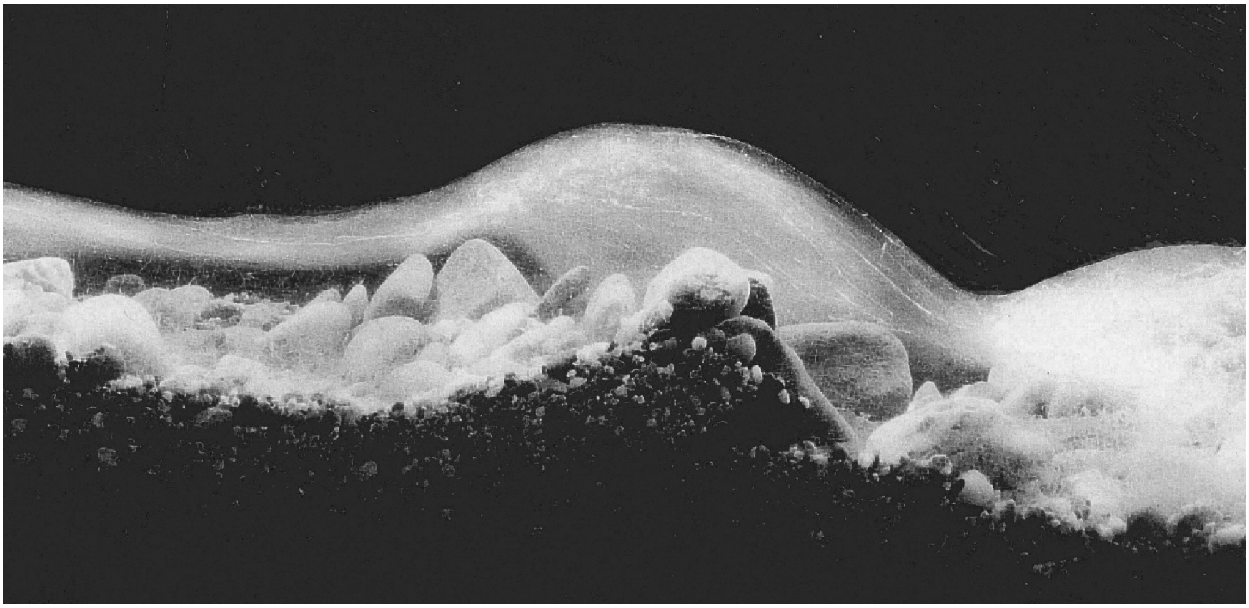

Figure 1 The flow over a step-pool structure. Photograph from Rosport [38].

nature in streams with low relative submergence and its precise determination is difficult.

Mountain streams have gravel to boulder sized beds and, in general, there are two different ways for data acquisition. The first is by surveying a specific cross sectional area and the water level. The cross-sectional mean velocity is estimated from point measurements with a current meter. Proceeding like this, the mean cross-sectional area and mean cross-sectional velocity are determined. Thus the discharge can be calculated with the equation of continuity. However, as mentioned for bed level datum and wetted perimeter, it is difficult to determine a representative cross section in mountain streams. Dependent on the location of the cross-section (e.g. step- or pool-section) different results for the mean velocity and the water depth will be obtained.

The second data acquisition method is by measuring the discharge and the mean flow velocity with the use of tracer methods, e.g. the salt-dilution method [22]. This method yields an estimate of a spatial and temporal averaged velocity over a channel section of a certain length. If the mean reach width is known, the reach averaged water depth can be calculated with the equation of continuity.

Generally, the determination of flow resistance requires the definition of a roughness parameter. Much attention has been devoted to developing a single, characteristic index of grain roughness d_c (derived from the grain size distribution of the surface material) that may be incorporated into calculations of flow resistance. However, a single grain-size gives only a coarse description of the roughness geometry, especially in the case of step-pool systems as indicated by Fig. 1. Information on the vertical extent of roughness or along the flow direction can not be explained by a grain-size [1,2,11]. Thus, the form resistance of the step-pool structures, which is undoubted also significant, remains largely unquantified [42]. Furthermore, flow resistance depends strongly on the density of roughness elements for conditions of low relative submergence. When the water depth is in the same order of magnitude as the roughness elements, the friction factor can vary by one order of magnitude depending on the roughness density [31].

This paper first reviews conventional approaches for determination of flow resistance in steep mountain streams. A new approach is developed by reviving an overlooked roughness parameter, the standard deviation of the bed elevations s [20,33]. The applicability of s to give a more accurate description of the geometry of the river bed will be shown and the superior performance of s as a hydraulic roughness parameter will be demonstrated with the use of experimental data.

2 Literature review of approaches

Existing approaches for the determination of flow resistance in mountain streams may be subdivided into three cases. These are either based on the logarithmic law of the wall, based on the Froude number or based on power-laws.

2.1 Logarithmic approaches

Several approaches to the quantification of flow resistance in gravel-bed streams have been based on boundary layer theory. These approaches are generally of the form:

$$\sqrt{\frac{8}{f}} = \frac{1}{\kappa}\ln\frac{h}{k_s} + B_r \tag{1}$$

with f = Darcy-Weisbach friction factor, κ = von Karman constant, h = mean water depth, B_r = constant, and k_s = equivalent sand roughness.

The value of the von Karman constant is normally given as $\kappa = 0.4$. However, for flows with low relative submergence Dittrich and Koll [16] found that a log-profile for the velocity profile fitted best with a value of $\kappa = 0.18$. This is also reported by Bayazit [9]. The required reduction in the value of κ may indicate some uncertainty in the theoretical applicability of Eq. (1) to flows with large relative roughness. The influence of bed-forms on the flow was not considered in the investigations of Dittrich and Koll [16] and Bayazit [9]. The parameter B_r generally has different values for different cross-sectional shapes [21,25] or for different relative submergence [26]. The equivalent sand roughness k_s is usually related to a particular percentile d_c of the grain-size distribution by a multiplying factor [6,15,18,21,29].

However, bed-forms, the wide range of bed materials, the three-dimensional and non-uniform nature of river flows and the deformation of the free surface violate the theoretical assumptions behind Eq. (1) and its application to low relative submergence conditions can be considered only on an empirical basis [7]. Resistance equations have been developed by adjusting the parameters κ, k_s and B_r to give the best fit to measured data. Proceeding like this, derived equations have a strong dependency on the data set which has been used and it is not surprising that there is a multitude of different equation coefficients. Further approaches have tried to include the influence of system-, form- and grain-roughness (e.g. [32]) but the equations still have a characteristic grain size as the basic roughness parameter. A special-case of these approaches will be discussed in the following section.

2.2 *Froude approaches*

A component of the form drag generated by large roughness elements in mountain streams is free surface drag which varies with the Froude number Fr and the relative submergence [8]. Several methods can be found in the literature which take the additional energy losses into account [5,8,10,12,14,23,38,39]. These incorporate the Froude number in order to determine the friction factor. However, for a given slope S, both f and Fr are calculated from the same set of parameters as shown by the relationship:

$$\sqrt{\frac{8}{f}} = \frac{\bar{u}}{u_*} = \frac{\bar{u}}{\sqrt{ghS}} = \frac{Fr}{\sqrt{S}} \qquad (2)$$

with $\bar{u}$ = mean velocity, u_* = shear velocity and g = acceleration due to gravity.

Thus, the results of approaches which seek to predict f directly from Fr, such as [5,14,38] may be based on spurious self correlation and should not be considered as robust predictors of flow resistance [1,3]. All equations based on approaches which determine the friction factor as a function of the Froude number must be treated with caution because of the intrinsic self correlation of Eq. (2).

2.3 *Power laws*

Another empirical way to derive a relationship for flow resistance or to establish a relationship between mean velocity and mean water depth (or the discharge) is by means of power laws. For a single river section with a constant slope S and a stationary bed (d_c = const.), the following relationship between the mean velocity and the discharge can be formulated:

$$\bar{u} = cQ^m \qquad (3)$$

with Q = discharge and c, m = constant factors.

A range of equations similar to Eq. (3) are linked by the regime concept [28] and Hydraulic Geometry concept [30], whereby relationships are formulated for the mean water depth h as well as the mean channel width w as a function of the discharge Q. The latter concept relates the constants of Eq. (3) and the at-a-site hydraulic geometry.

According to Bathurst [7] the exponent m is of particular interest. It indicates the rate of change of velocity and adopts certain characteristic values. Where there are large changes in flow resistance between high and low flows, the exponent may be expected to be relatively high. This applies to step-pool streams. At low flows, ponding has a much greater effect on flow resistance than bed material roughness. For these conditions ponding is the dominant cause of resistance. As the ponding effect is drowned out at high flows, bed material roughness becomes the dominant cause of resistance. Consequently, the overall reduction in resistance is larger compared to the scenario when only bed material roughness is important. According to Bathurst [7], the exponent m increases in value, moving up a channel network from sand bed channels ($m < 0.40$) via gravel ($m = 0.45$–0.55), cobble and boulder-bed channels ($m = 0.45$–0.55) to steep pool/fall streams and pool-riffle sequences ($m > 0.55$).

A similar interpretation concerning the exponent m was given by Kellerhals [24] who suggested that the hydraulic character of tumbling flow channels moves with increasing slope and decreasing channel width gradually from uniform open channel flow towards flow in a cascade of parabolic weirs. His estimates of m from experimental data lie within the limits of $0.4 \leq m \leq 0.714$, with the value of $m = 0.4$ derivable from Manning's formulae. However, the limits for the exponent given by Kellerhals [24] can be enlarged on by theoretical considerations. Inserting Eq. (3) into the definition of the Froude number $Fr = \bar{u}w^{0.5}/(gA)^{0.5}$, where A is the cross sectional area and w is the water surface width [13] and using the equation of continuity yields:

$$Fr = c^{1.5}w^{0.5}g^{-0.5}Q^{1.5m-0.5} \qquad (4)$$

On the basis of Eq. (4), limits for the exponent m can be derived as a function of the cross-sectional shape. E.g. assuming a rectangular channel cross-section (w = const.), the first derivative of Eq. (4) with respect to the discharge Q gives:

$$\frac{dFr}{dQ} = c^{1.5}w^{0.5}g^{-0.5}(1.5m - 0.5)Q^{1.5m-1.5} \qquad (5)$$

Reasonably assuming that the Froude number does not decrease with increasing discharge, i.e. $dFr/dQ \geq 0$, Eq. (5) gives $m \geq 1/3$ as the lower limit for m for a rectangular cross-section. The lower limit for a parabolic and a triangular cross-section can be calculated as 1/4 and 1/5 respectively. If m is smaller than this lower limit, the Froude number must decrease with increasing discharge. Substituting $m = 1$ in Eq. (5) results in a constant value for dFr/dQ, i.e. a linear change of Froude number with discharge. Furthermore, inserting $m = 1$ into Eq. (3) yields $Q/\bar{u} = A = 1/c$. Thus for $m = 1$ the cross-sectional area does not change with varying discharge, independent of the cross-sectional shape. A value of m larger than this upper limit of $m = 1$

would mean that the flow cross-sectional area must decrease with increasing discharge and this is not possible.

Most of the existing approaches following Eq. (3) have not been restricted to only one cross section. These regime equations transform into power law flow resistance equations by introducing the slope and a roughness parameter d_c. Proceeding like this, Eq. (3) can be written:

$$\bar{u} = c_1 g^{m_1} S^{m_2} Q^m d_c^{m_3} \tag{6}$$

with c_1 = an empirical constant.

Approaches following Eq. (6) are dimensional homogeneous if the relationships $m_1 = (1 - m)/2$ and $m_3 = (1 - 5m)/2$ are fulfilled. Results of this type of approach range from that of Rickenmann [36, Eq.(7)] to those of Rickenmann [37, Eq.(8)]:

$$\bar{u} = 1.3 g^{0.20} S^{0.20} q^{0.60} d_{90}^{-0.40} \qquad 3\% < S < 40\% \tag{7}$$

$$\bar{u} = 0.37 g^{0.33} S^{0.20} Q^{0.34} d_{90}^{-0.35} \qquad \text{for } S > 0.8\% \tag{8a}$$

$$\bar{u} = 0.96 g^{0.36} S^{0.35} Q^{0.29} d_{90}^{-0.23} \qquad \text{for } S < 1.0\% \tag{8b}$$

with q = discharge per unit width.

Note that according to the above considerations, Eq. (8b) should not be applicable to a rectangular cross-section since the exponent of $m = 0.29$ is smaller than the lower bound of the exponent for a rectangular cross section of 1/3.

To summarise, the existing logarithmic, Froude and power law approaches for the determination of flow resistance are based on the assumption that streams with the same slope and the same characteristic grain diameter show exactly the same hydraulic behaviour. Some shortfalls of these approaches were discussed. We contend that streams characterised by the same roughness height but a different arrangement of the roughness elements can show different hydraulic behaviour. This is now investigated with the help of experimental data.

3 Data

The following analysis uses experimental data sets from the Theodor-Rehbock laboratory at Karlsruhe University from Rosport [38] and Koll [27]. Both sets of experiments were carried out in a 0.20 m wide, 0.30 m deep and 6.8 m long tilting flume with flume slopes of 2, 4, 8 and 9.8%. Two coarse sediment mixtures with 1 mm $< d <$ 64 mm and 1 mm $< d <$ 32 mm were used as movable bed material for the development of water worked roughness structures. In the experiments, the discharge Q was adjusted by a valve and measured by an inductive flow meter with an accuracy of ±0.02 l/s [38]. The mean flow velocity $\bar{u}$ was measured by the salt dilution method over a 2.40 m long test section. From this data, the reach averaged water depth was calculated as $h = Q/(\bar{u}w) = q/\bar{u}$.

In the first step of the experiments the appropriate, well mixed sediment was placed in the flume and flattened to a thickness of 15 cm. The slope of the surface was parallel to the slope of the flume. The surface was then allowed to armour and a degree of bed stability was established for a given discharge $Q_{\max 1}$. At this stage of armouring, mean flow velocities were determined for $Q_{\max 1}$ and lower discharges. After the discharge was reduced to zero, photographs of the armour layer were taken and analysed by applying a line by number method. The correction method of Fehr [17] was used to enable a comparison of the results with volumetric samples.

The bed topography was measured along the 2.40 m long test section with a laser displacement meter with a sample interval of 2.4 mm. Profiles in flow direction were recorded with a lateral spacing of 10 mm and, depending on the experiments, 10–16 parallel longitudinal profiles were recorded. The precision of the laser probe is 0.06 mm in the y and z-direction and 0.5 mm in the x-direction.

After surveying the bed surface, the bed forming discharge $Q_{\max 1}$ was increased for the next step to a value $Q_{\max 2}$ ($> Q_{\max 1}$), so that the existing armour layer was destroyed and a new one developed. The above measurements were repeated and the procedure continued as long as the stream could stabilise itself without a considerable loss of slope or the sediment was eroded to the flume-bottom. Data from Koll [27] were obtained in a slightly different way. Dependent on the experiments, different sediment fractions were fed at different feeding rates and different discharges after the first stabilisation at $Q_{\max 1}$. After the feeding, the surface armoured again and the same procedure was repeated as in the experiments of Rosport [38], i.e. the discharge was increased again etc. More details concerning the measuring techniques and data acquisition methods are found in Rosport [38], Aberle [1] and Koll [27].

The mean bed slope for each run was obtained from a linear regression of the laser surface elevation data. The mean slope was first calculated for each profile before the ensemble average bed slope was calculated from the 10–16 profile slopes. We note that linear regression is the simplest way of obtaining the average bed slope for beds with step-pool systems. For rivers where bed material roughness is the main source of resistance it may be appropriate to detrend the data using non-linear methods (e.g. bicubic spline interpolation) to remove channel slope and any large scale distortion in the surface of the bed [41]. However, in the case of step-pool systems, form resistance is a major contributor to flow resistance. Thus we decided to derive further statistical parameters directly from the linear detrended profiles, which are influenced by both the microtography of the bed and step heights [1,29]. As for the slope, the statistical parameters were first calculated for each detrended profile before the ensemble average was calculated. Information on the experiments, such as flume slopes, sequential bed-forming discharges $Q_{\max}$, the number of velocity measurements for each sequential run (for discharges equal and smaller than $Q_{\max}$), the bed material and estimated roughness parameters (mean grain-diameter $d_{m,D}$ and standard deviation s) for each stable armour layer is summarised in Table 1.

4 Geometrical description of the surface structure

Figure 2 shows two detrended profiles of beds which are characterised by the same mean grain diameter $d_{m,D}$. In the figure, the

Table 1 Hydraulic data and roughness parameters.

Experiment[a]	Slope[b] [%]	Bed forming discharges $Q_{\max,i}$[c] [l/s]	No. of velocity measurements[d]	$d_{\max}$ [mm]	Mean grain diameter $d_{m,D}$[e] [mm]	Standard deviation s[f] [mm]
R1	8–10	2.81, 4.09, 4.15, 4.84, 5.43	$4^V, 5^V, 6^D, 9^V, 6^D$	64	32.2, 32,8, 31.6, 34.4, 33.6	11.1, 11.7, 12.5, 12.8, 11.4
R2	8–10	2.76, 3.44, 4.27, 4.77, 5.37	$5^D, 5^V, 6^V, 7^D, 7^D$	64	32.5, 36.0, 30.9, 36.0, 36.6	10.8, 11.6, 12.7, 12.4, 12.2
R3	8–10	1.70, 2.30, 2.90, 3.07	$3^D, 4^D, 6^V, 6^V$	32	16.1, 17.1, 17.8, 17.6	6.4, 8.3, 9.1, 8.4
R4	8–10	2.54, 2.82, 3.10	$4^D, 5^V, 5^D$	32	16.7, 18.1, 17.0	7.9, 7.6, 8.0
R5	2	7.22, 8.80	$7^V, 6^D$	32	15.8, 16.7	5.0, 5.4
R6	2	9.73, 12.49, 14.28, 16.15	$6^D, 7^D, 6^V, 6^D$	32	16.0, 16.4, 16.6, 17.1	6.1, 7.2, 6.6, 6.9
R7	2	4.37, 8.13, 11.03, 14.15, 16.11	$3^D, 5^V, 6^V, 6^V, 6^D$	32	12.9, 15.3, 16.1, 17.0, 15.8	4.6, 5.4, 6.4, 7.6, 6.2
R8	4	3.42, 4.63, 5.47, 6.41, 7.09	$4^D, 5^V, 6^D, 6^D, 7^V$	32	16.2, 16.8, 16.5, 17.6, 16.4	5.7, 6.0, 7.4, 7.1, 7.03
R9	4	3.55, 4.60, 5.62, 6.37, 6.96	$4^V, 5^D, 6^V, 6^D, 6^V$	32	n/a, 15.8, 16.8, 16.2, 15.8	5.5, 7.1, 6.8, 7.1, 6.3
R10	4	4.61, 5.48, 6.41, 7.45, 8.34	$5^D, 5^D, 6^V, 6^V, 8^D$	64	29.3, 26.5, 24.8, 27.9, 28.1	7.0, 7.5, 8.5, 10.0, 9.7
R11	4	4.68, 5.49, 6.29, 7.83, 8.03	$4^D, 5^V, 6^V, 6^D, 7^V$	64	24.5, 28.7, 28.7, 31.3, 32.3	7.1, 8.1, 9.3, 11.3, 10.7
R12	2	6.16, 7.69, 12.64, 14.96, 17.59	$5^V, 6^V, 6^D, 7^D, 7^V$	64	19.6, 20.8, 27.4, 23.6, 25.3	4.6, 5.8, 7.3, 8.2, 7.5
R13	2	7.96, 9.96, 12.72, 15.23, 17.49	$6^D, 6^V, 6^D, 6^D, 6^V$	64	26.0, 29.7, 33.2, 31.5, 33.4	6.1, 6.7, 8.2, 9.6, 8.7
K1	8–10	3.15, 3.10, 4.16, 4.60, 5.60, 6.20	$5^D, 5^V, 7^D, 6^V, 7^D, 10^D$	64	26.4, 27.5, 27.3, 28.0, 25.4, 30.2	9.6, 9.0, 12.3, 13.9, 14.0, 14.6
K2	8–10	3.10, 3.20, 4.10	$5^D, 5^D, 7^D$	64	29.0, 26.1, 28.5	9.5, 9.9, 11.5
K3	8–10	3.10, 3.05, 3.49	$5^V, 5^D, 6^D$	64	26.0, 17.0, 19.7	10.9, 8.2, 9.3
K4	8–10	3.00, 2.91, 3.42	$5^V, 5^V, 6^D$	64	23.0, 26.0, 29.6	9.6, 10.6, 11.5
K5	8–10	3.07, 2.94, 3.98	$5^D, 5^V, 7^D$	64	28.7, 30.4, 32.1	12.0, 12.4, 12.4
K6	8–10	2.96, 3.04, 3.02, 3.69	$5^V, 5^V, 5^D, 6^V$	64	28.6, 26.6, 26.5, 31.8	10.8, 12.7, 11.6, 10.9
K7	8–10	3.10, 3.14	$5^D, 5^D$	64	27.8, 28.9	11.2, 10.2
K8	8–10	3.02, 2.85	$5^V, 5^V$	64	26.4, 33.6	10.1, 10.0
K9	8–10	3.05, 2.99, 3.50, 4.15	$5^D, 5^V, 6^V, 7^V$	64	29.0, 26.0, 28.0, 36.2	12.6, 10.2, 10.4, 11.8
K10	8–10	3.08, 3.13, 3.63	$5^V, 5^V, 6^D$	64	30.4, 27.0, 27.9	11.5, 11.9, 12.4
K11	8–10	2.42, 2.38, 3.11	$4^D, 4^V, 5^V$	64	25.4, 28.9, 30.2	8.9, 10.0, 10.2

[a] R: Experiments of Rosport [38], K: Experiments of Koll [27]; [b] Flume slope. Note that the bed slope was determined for each experiment from laser data; [c] Sequential bed forming discharges $Q_{\max,i}$ according to the experimental procedure. Note that for the experiments of Koll [27] sediment of different fractions was fed at different feeding rates after the first stabilisation; [d] Number of velocity measurements for the bed forming discharge $Q_{\max,i}$ (column 3) and discharges smaller than bed forming discharge ($Q < Q_{\max,i}$) over the same static surface; [D] Data used for the derivation of the formulae in Section 5; [V] Data used for the validation of the derived formulae; [e] Mean grain diameter of the armour layer formed by the bed forming discharges given in column 3; [f] Ensemble average of standard deviation (average value) for the armour layer formed by the bed forming discharges given in column 3.

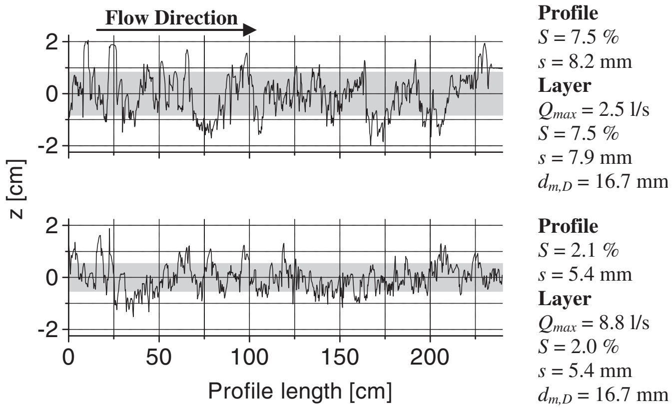

Figure 2 Two single laser scan profiles of the flume bed for two different runs. Profiles values are shown as well as the layer values (calculated as ensemble average of all measured profiles).

average values of the characteristic parameters of all profiles are given as well as the slope and the standard deviation of the bed elevations for the single profiles. Although being characterised by the same $d_{m,D}$, the upper profile in Fig. 2 shows clearly a more extensive roughness structure than the lower profile due to different flow history (different slope and bed forming discharge). In the upper profile step-pool systems occur, whereas such structures cannot easily be identified in the lower profile. However, in contrast to the $d_{m,D}$, the difference in the roughness structure can be described by the standard deviation of the bed elevations s. This parameter can easily be derived if longitudinal profiles scanned with a high resolution are available. The standard deviation of the upper profile is higher ($s = 8.2$ mm) than for the lower profile ($s = 5.4$ mm), reflecting the obvious differences in the roughness structure. This is indicated on Fig. 2 by the grey area which shows an interval of $0 \pm s$. If the ensemble average s-values for the 10 scanned profiles for each layer are compared with each other, the same result is obtained (see Fig. 2).

This result can be explained as follows. In determining a grain size distribution, the full extent of the grains is taken into account. A characteristic grain diameter is selected, normally representing the larger grains. These grains are embedded into the substratum and do not contribute their full size to the surface roughness. The peak values in the bed elevations that result e.g. by step-pool systems (see upper profile, Fig. 2) are not considered when the grain-size distribution is determined. These peaks are considered when s is calculated.

Figure 3 shows a comparison of the roughness parameters $d_{m,D}$ and s which are given in Table 1. Note that the mean bed slope was determined according to the procedure described above and is not identical to the flume slope. Within the conducted experiments no significant increase of the mean diameter $d_{m,D}$ with the mean bed slope can be identified for a given bed material, as shown in Fig. 3a. The dependency on type of bed material is due to the different maximum grain-diameters of 64 and 32 mm of the sediment mixtures used. In contrast to the mean diameter $d_{m,D}$, the standard deviation s increases with increasing slope, as shown by Fig. 3b. The dependency on type of the bed material is found as well in Fig. 3b, but not as distinctive as in Fig. 3a. The largest values for s have been obtained for layers with step-pool systems.

The scatter of the s values for the same $d_{m,D}$ in Fig. 3c confirms that beds armoured with the same $d_{m,D}$ can show a different roughness structure. Figure 3c also shows that armour layers with a different $d_{m,D}$ can be characterised by the same s, indicating a similar roughness structure of these beds [1,2]. Furthermore, no significant correlation between s and $d_{m,D}$ can be identified

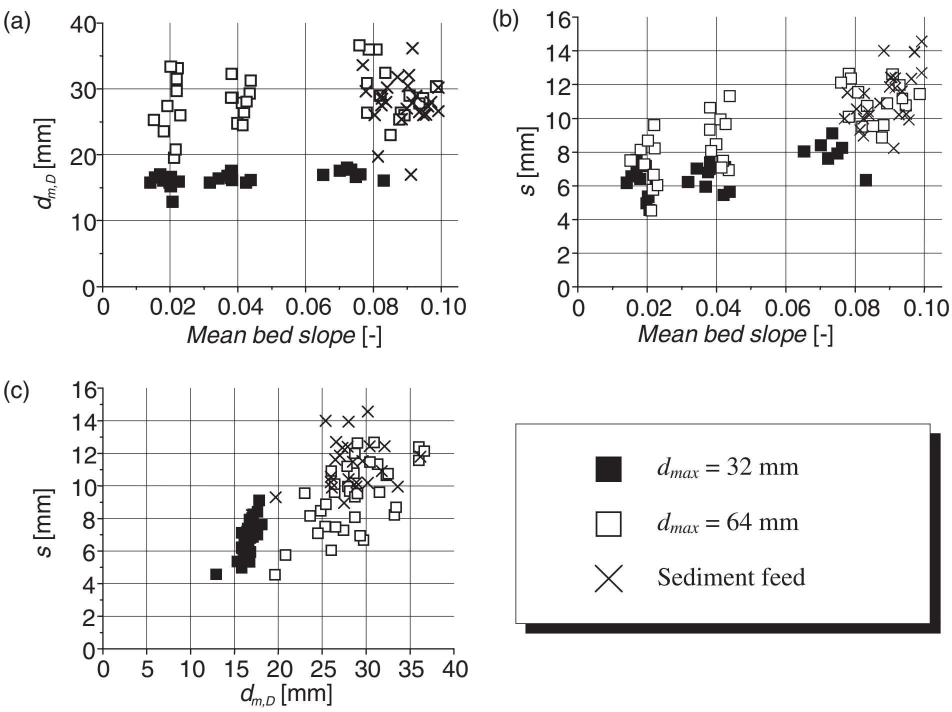

Figure 3 (a) The mean diameter of the armour layer $d_{m,D}$ as a function of the mean bed slope. (b) The standard deviation s as a function of the mean bed slope. (c) The standard deviation s as a function of the mean diameter $d_{m,D}$ of the armour layer.

except when the aforementioned dependency on the type of bed material is taken into account.

5 Flow resistance and standard deviation

Due to the abundance of hydraulic data, the data were randomly distributed into two groups for the following analysis, an investigation and a validation data set (see Table 1). The variability in the slope and bed material was chosen to be equal in both data sets. This means that half of the experiments with $d_{\max} = 64$ mm and $d_{\max} = 32$ mm were randomly placed in the investigation and half in the validation set.

Figure 4a shows the plot of $(8/f)^{0.5}$ against h/d_{84} for the investigation data. As noted in Section 3, the reach averaged water depth h was calculated as $h = q/\bar{u}$. Figure 4a shows that the data used in this study cover ratios of $h/d_{84} < 3$ and that $(8/f)^{0.5}$ increases with increasing h/d_{84}. Fitting the investigation data shown in Fig. 4a to a relationship following Eq. (1) by a least square method yields the following empirical relationship:

$$\sqrt{\frac{8}{f}} = 3.54 \ln\left(\frac{h}{d_{84}}\right) + 4.41 \tag{9}$$

A comparison of the measured and calculated values of $(8/f)^{0.5}$ with the validation data is shown in Fig. 4b ($R^2 = 0.80$). However, Fig. 4a and b show a distinct dependency on the type of sediment mixture used. It is postulated that this scatter is due to the fact that the geometric surface structure cannot be described adequately by a characteristic grain size.

Using the standard deviation s instead of d_{84} as a roughness parameter, the plot of Fig. 5a is obtained for the investigation data set. In Fig. 5a, the dependency on the bed material disappears and the scatter of the data is greatly reduced. With the use of the data shown in Fig. 5a and a least squares fitting procedure, the relationship between $(8/f)^{1/2}$ and h/s can be expressed in terms of a log-law as:

$$\sqrt{\frac{8}{f}} = 3.86 \ln \frac{h}{s} - 1.19 \tag{10}$$

For the comparison of calculated and measured values of $(8/f)^{0.5}$, shown in Fig. 5b for the validation data, a R^2-value of 0.91 is obtained. This R^2-value is higher than the one obtained for Eq. (9) and indicates the improved fit of the data. However, as mentioned above, approaches based on the log-law in mountain streams are strongly empirical, as is Eq. (10).

Based on a spatially-averaged description of the flow field Nikora *et al.* [35] concluded that for flow where the water level is near to the tops of the roughness elements (for ratios $h/d_{84} < 5$, which applies to the data of this study and for steep mountain streams in general), the friction factor f is proportional to $(\delta/h)^2$, with δ = height of the roughness layer. This relationship was postulated because the flow above the roughness elements is heavily influenced by eddies of scale δ generated in the wakes of roughness elements. The original analysis in Nikora *et al.* [35] assumed $\delta = d_{84} \approx d_{90}$, thus a straight line would be expected in Fig. 4a.

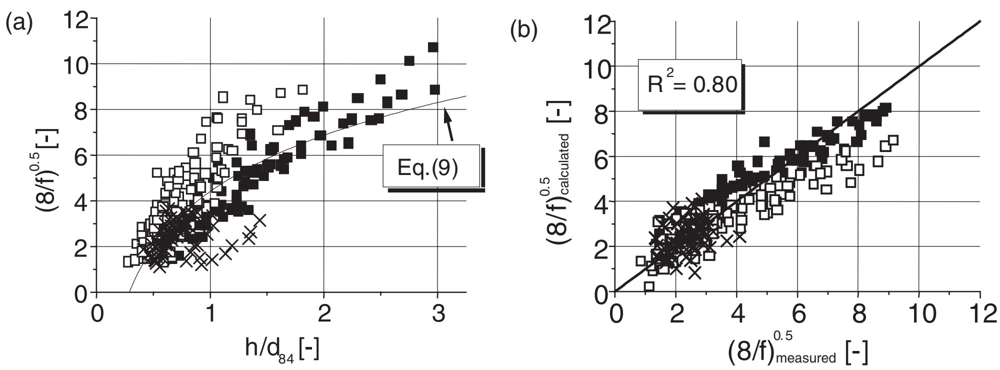

Figure 4 (a) Plot of $(8/f)^{0.5}$ against h/d_{84} for the derivation data and (b) validation of Eq. (9) by comparing $(8/f)^{0.5}_{\text{calculated}}$ and $(8/f)^{0.5}_{\text{measured}}$ with the validation data set. Legend see Fig. 3.

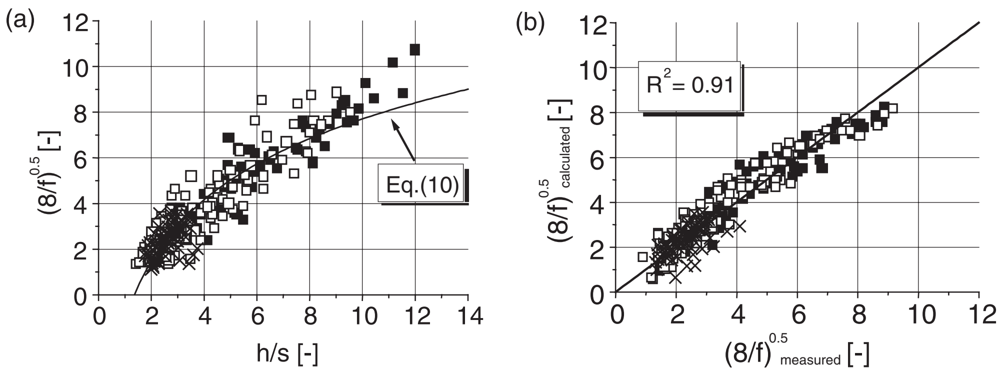

Figure 5 (a) Plot of $(8/f)^{0.5}$ against h/s for the derivation data and (b) validation of Eq. (10) by comparing $(8/f)_{\text{calculated}}$ and $(8/f)^{0.5}_{\text{measured}}$ with the validation data set. Legend see Fig. 3.

Here, the applicability of this description of the flow field is tested for rough surfaces which included bed forms (step-pool systems). As Fig. 4a shows, a dependency on the bed material would be introduced for this data by assuming $\delta = d_{84}$. The reduced scatter of the data in Fig. 5a shows that the standard deviation is a better measure for this layer thickness, i.e. $\delta \propto s$. The best fit relation of this type is:

$$\sqrt{\frac{8}{f}} = 0.91\frac{h}{s} \tag{11}$$

Compared to Eq. (10) a slightly improved fit of the data is obtained ($R^2 = 0.92$), since the data points for the ratio $8 < h/s < 12$ are fitted more accurately by Eq. (11) than by the logarithmic fit of Eq. (10).

As mentioned, the determination of the mean water depth is difficult in steep streams and Eq. (11) may be rearranged into the form of Eq. (6). Taking into account that the flume width was constant during the conducted experiments, the use of the equation of continuity, Eqs. (2) and (11) yields:

$$\bar{u} = 0.96 g^{0.20} S^{0.20} q^{0.60} s^{-0.40} \tag{12}$$

Figure 6 shows a comparison of the measured and calculated mean velocities for the validation data. The obtained regression coefficient of $R^2 = 0.98$ proves again the goodness of the fit of the derived relationship. Comparing Eq. (12) with Eq. (7) shows the same exponents for discharge and slope, i.e. a similar relationship as proposed by Rickenmann [36]. The only difference between Eqs. (12) and (7) is that the roughness parameter d_{90} is replaced by the standard deviation of the bed elevations.

A further data analysis technique is to investigate dimensionless groupings of the quantities $\bar{u}, q, S, g, d_c$ and s. If Reynolds-number effects are neglected, this yields the relationship $\bar{u}/(gd_c)^{0.5} = f(q/(g^{0.5}d_c^{1.5}), S, s/d_c)$ and a nonlinear multiple regression with the derivation data gives $\bar{u} = 1.06 g^{0.18} S^{0.26} q^{0.64} s^{-0.46}$ which is essentially the same as Eq. (12). Comparing the obtained relationship with the functional relationship from the dimensional analysis shows that the regression fit has eliminated the grain size. This result is obtained, independent of whether the grain diameter is taken as $d_{m,D}$ or d_{84}.

In deriving the foregoing relationships, the hydraulic geometry is not taken into account and the effect of the step-pool systems (ponding) on flow resistance remains unclear. However, the way the experiments were conducted (velocity measurements for various discharges for each armour layer, Table 1) made it possible to investigate the at-a-site hydraulic geometry in detail. Power law relationships ($\bar{u} \propto q^m$) were derived by linear regression in the log-log space using the dimensional homogeneous equation $\bar{u}/(gs)^{0.5} = f(q/(g^{0.5}s^{1.5}))$. Each derived relationship consists of at least 3 pairs of measurements for the mean velocity and the discharge over the same bed structure (Table 1). In each of the 94 cases R^2 values near to 1 were obtained. Note that we waived the subdivision of the data into an investigation and validation set for this analysis.

This strategy allows the influence of the slope on the exponent m to be investigated. Figure 7 shows the plot of the obtained exponents as a function of the slope and exhibits a general increasing trend of the exponent with increasing slope. For bed slopes of 2%, the lowest values of the exponent m are observed. Wide scatter is obtained for the slopes of 8–9.8%. Generally, the obtained values for m are in agreement with the range of the exponent given by Bathurst [7]. The scatter of the exponents for the high slopes is attributed to the variable bed morphology. On the assumption that a large exponent indicates a large roughness of the riverbed and/or a large volume of the step-pool systems, Fig. 7 indicates that the geometry of these structures can deviate remarkably from each other.

The exponents of $m = 0.60$ and $m = 0.64$ given by Eq. (12) and obtained by the dimensional analysis procedure respectively, are indicated in Fig. 7. The plot shows that the influence of the slope could be underestimated if a constant exponent is used in Eq. (3). An empirical approach for the determination of flow resistance based on Fig. 7 did not yield better results than the above approaches [1].

These findings show that further research is needed in the classification of step-pool systems, especially concerning their geometry and their influence on hydraulic resistance. So far only a description of the step-pool systems concerning the ratio of step length to step height as a function of the bed slope is found in the literature [4]. The derivation of a roughness length is discussed by Smart *et al.* [41] and Nikora *et al.* [34] for gravel-bed rivers, but no reliable information on the spacing of the roughness in flow direction is yet available for the case of steep streams. Certainly, to be able to derive a detailed roughness model of

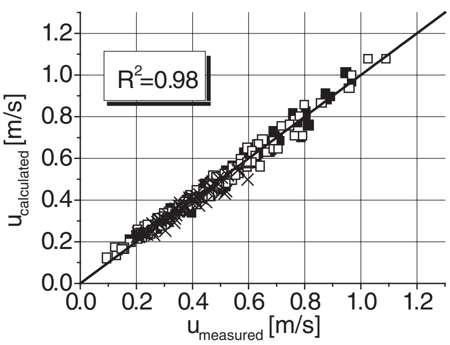

Figure 6 Comparison of calculated and measured velocity following Eq. (12) for the validation data. Legend see Fig. 3.

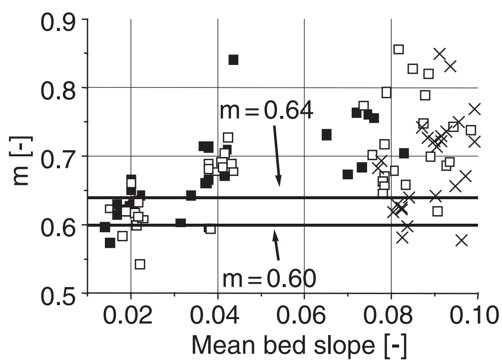

Figure 7 The exponent m as a function of the slope. The solid lines show the constant exponents of $m = 0.60$ and $m = 0.64$ respectively. Legend see Fig. 3.

streams, this characteristic roughness length is a crucial piece of information. Further investigations of longitudinal bed profiles of steep mountain streams by statistical and spectral analyses should focus on the identification of different roughness scales. Thus a differentiation of the influence of skin- and form roughness on hydraulic resistance in mountain streams should become possible and, finally, the scatter of the data in Fig. 7 may explainable.

6 Summary and conclusion

An analysis of the log-law and power law resistance equations show that these approaches have theoretical shortcomings when applied to streams in which the roughness elements are of the same order of magnitude as the water depth. Additionally the effect of bed forms on flow resistance has not been included in most investigations. It is common to use a characteristic grain size for the roughness parameter and this is usually derived from the grain size distribution of the bed material.

The investigations of [31] and others show that the geometric arrangement of roughness elements is a significant factor for the determination of flow resistance in the case of flow with small relative submergence. In particular, for flow over rough gravel- and boulder-bed surfaces, it can be concluded that hydraulic roughness can change with different bed surfaces even though they are characterised by the same roughness parameter d_c.

The main goal of this investigation was to find a roughness parameter which is able to give a better description of the roughness structure of rough beds. Analysis of longitudinal bed profiles from laboratory experiments identified the standard deviation s of the bed elevations as a more appropriate roughness parameter. The effect of channel slope was removed but s still includes the effect of morphological structures, such as step-pool sequences. These features are not considered when a characteristic grain size is determined from an analysis of the bed material.

It was found that the use of the standard deviation s as a roughness parameter leads to an improvement in estimation of flow resistance compared to the results obtained by using only d_c as roughness parameter. For the laboratory data investigated, Eq. (12) was found to give the best prediction of flow velocity. It was shown that Eq. (12) complies with spatial averaging of the flow field according to the approach of Nikora *et al.* [35], and that the flow resistance equation of Rickenmann [36] is essentially the same as Eq. (12) with the exception that the earlier equation uses grain-size in place of standard deviation. Furthermore, when the standard deviation was included it was found that the derived equation could not be improved by inclusion of any grain-size parameter. This shows the pertinence of the standard deviation as a roughness parameter.

In order to study the influence of the slope, which changes the geometry of step-pool systems the data was additionally analysed empirically by considering at-a-site hydraulic geometry. Based on 94 relationships of the form $\bar{u} \propto q^m$ for the flow over stable armour layers it was concluded that the exponent m varies as a function of the slope and the local site properties (e.g. bed geometry, etc.). Though good results have been obtained, a constant exponent of $m = 0.60$ cannot take these variations into account. Therefore, further research should focus on deriving a characteristic roughness length in the flow direction to be able to derive a more detailed roughness model.

Field data is needed for the purpose of validation of the findings of this study. Due to the experimental procedure, the influence of the stream width was neglected. Furthermore, Bathurst [7] states, that investigations in laboratories yield values of the exponent m, which are lower than exponents for natural streams. Although the bed surface consists of materials characterised by a certain grain-size distribution, for the two sediment mixtures studied, no relationship could be derived between a characteristic grain-size and the standard deviation of the bed elevations. Further studies should be made to investigate the relationship of the slope, grain-size distribution and the surface standard deviation for field data, especially for different sites and different streams.

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Notations

A	=	cross sectional area
B_r	=	constant
c, c_1	=	constant
d_c, d_{84}, d_{90}	=	characteristic grain-size
$d_{m,D}$	=	mean diameter of the armour layer
$d_{\max}$	=	maximum grain diameter of the mixture
f	=	Darcy-Weisbach friction factor
Fr	=	Froude Number
g	=	acceleration due to gravity
h	=	mean water depth
k_s	=	equivalent sand roughness
m, m_1, m_2, m_3	=	exponents
Q	=	discharge
q	=	specific discharge per unit width
S	=	bottom slope
s	=	standard deviation of the bed elevations
u_*	=	shear velocity
$\bar{u}$	=	mean flow velocity
w	=	channel width
δ	=	height of roughness layer ($\approx d_{84}$)
κ	=	von Karman constant

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