MODELLING OF STEADY TRANSCRITICAL FLOW IN OPEN CHANNELS

YEBEGAESHET T. ZERIHUN [1] and JOHN D. FENTON [2]

[1] Graduate Student, Department of Civil & Environmental Engineering,
The University of Melbourne, Victoria □010, Australia
(Tel: +61-□-8□44-7178, Fax: +61-□-8□44-4616, e-mail: y.zerihun@civenv.unimelb.edu.au)
[2] Professorial Fellow, Same Institute
(Tel: +61-□-8□44-9691, Fax: +61-□-8□44-4616, e-mail: fenton@unimelb.edu.au)

Abstract

A numerical experiment is carried out to investigate the suitability of a one-dimensional Boussinesq-type model for simulating transcritical flow in a channel with simultaneous variation of sidewalls and bottom geometries. Two Boussinesq-type momentum equation models for such type of flow simulation are considered in this study. These models incorporate different degrees of dynamic pressure corrections as a result of the pre-assumed uniform and linear variation of centrifugal term at a vertical section. The effect of the pressure correction factors on the solutions of the models is also examined. The finite difference method is employed to discretise and solve the flow equations. The models are then applied to simulate different test cases for flow in such channel with predominant non-hydrostatic pressure distribution effects. A comparison of the computed results with the corresponding experimental data is presented. Generally speaking, good agreement is attained between the computed and measured results from this comparison. Results of the study reveal that the 2-D flow structures for such type of flow situation are better described by the proposed model which includes a higher-order correction for the effect of the centrifugal pressure.

Keywords: Open channel flow; Hydrodynamics; Boussinesq equation; Nonuniform flow

1. INTRODUCTION

In most open channel flow measuring structures such as venturi flumes, the geometries of both the sidewalls and the bed vary along the direction of the flow. It is well-known that such geometric changes result in flow transition from subcritical to supercritical state under free flow conditions. The significant characteristic of the flow in the vicinity of such transition is a strong departure from the hydrostatic distribution of pressure caused by sharp curvatures of streamlines. The common computational flow models, which assume hydrostatic pressure distribution, do not retain accuracy for such types of flow situations. The essential vertical flow details of the flow situations require the use of relatively accurate methods for exact description of the flow problems. In this study, a one-dimensional model that incorporates a correction for the effect of the centrifugal dynamic pressure due to the curvature of the streamline will be employed for the numerical simulation of free flow in such types of flow measuring structures.

Several investigations have been performed in the past to model flow situations that involve non-hydrostatic pressure distribution effects using different approaches. Boussinesq (1877) was the first to extend the momentum equation to incorporate implicitly the streamline curvature effects. Recently, Dressler (1978), Hager and Hutter (1984), Hager (1985) and Matthew (1991) have developed higher-order equations to model two-dimensional flow problems. Similar to the Boussinesq (1877) equation, however these equations are limited to the solution of irrotational flow problems only. For fast flow over the chute part of hydraulic structures such as spillways, the assumption of irrotational flow, which was the basis of all these governing equations, is questionable. It has also been shown by Fenton (1996) that the scaling of the variables introduced into the Dressler equations has an inconsistency. Steffler and Jin (199□) developed the vertically averaged and moment (VAM) equations based on the assumptions of a linear longitudinal velocity distribution, and quadratic pressure and vertical velocity distributions across the depth of flow. However, the resulting equations are very long and complex. Khan and Steffler (1996a, b) studied the applicability of these equations. Fenton (1996) introduced alternative equations to model flow problems with appreciable curvature of streamline. He used the momentum principle along with the assumption of a constant centrifugal term, $\kappa / \cos\alpha$ (κ = curvature and α = angle of inclination of the streamline with the horizontal axis), at a vertical section to develop the equations. The distribution shape of this term determines the degree of the resulting correction factor for the effects of the streamline curvature. Compared to other governing equations (for instance, Dressler and VAM equations), the equations are simple to apply in a cartesian coordinate system especially for flow problems with continuous flow boundaries. In this study, these equations along with the modified version, which are developed based on the assumption of a linear variation of centrifugal term with depth, will be employed to simulate steady transcritical flow situation in flow measuring structures. These include free flows over trapezoidal profile weirs as well as in venturi flumes with and without humps.

Flow problems related to open channel flow measuring structures have been extensively studied experimentally. Most of the experimental works were performed to understand the flow characteristics of these structures as well as for the determination of the coefficients of discharge (see e.g., Bos, 1978). However, little attempt has so far been made to model such flows numerically using a higher-order flow equation. From a practical perspective, accurate simulation of the flow particularly the upstream water surface elevation and/or piezometric head is very important in order to predict the discharge capacity of these structures under free flow conditions.

Therefore, the purposes of this paper are: i) to investigate the Boussinesq-type equations models for simulating transcritical flow situation in a channel with simultaneous variation of sidewalls and bed geometries; and ii) to assess systematically the impact of the pressure correction factors on the simulation of pressure and flow surface profiles of such flows.

2. GOVERNING EQUATIONS

As discussed above, Fenton (1996) presented a simple method that incorporates the possible variation of the channel width. For steady flow in a rectangular channel, this method yields the following equations:

$$\beta\frac{Q^2H}{4A}\frac{d^3H}{dx^3}+\beta\frac{Z_b^{'}Q^2}{2A}\frac{d^2H}{dx^2}+\left(1+Z_b^{'2}\right)\left(\left(gA-\beta\frac{Q^2B}{A^2}\right)\frac{dH}{dx}+gA\left(Z_b^{'}+S_f\right)\right)$$
$$+\omega_0\beta\frac{HQ^2}{A}\left(\frac{Z_b^{'''}}{2}+\frac{Z_b^{'}Z_b^{''}}{H}\right)-\beta\frac{HQ^2}{A^2}\left(\left(1+Z_b^{'2}\right)+H\left(\frac{1}{2}\frac{d^2H}{dx^2}+\omega_0Z_b^{''}\right)\right)\frac{dB}{dx}=0, \tag{1}$$

$$p=\rho(\eta-z)\left(g+\frac{\beta Q^2}{A^2\left(1+Z_b^{'2}\right)}\left(\omega_0Z_b^{''}+\frac{1}{2}\frac{d^2H}{dx^2}\right)\right), \tag{2}$$

in which H is the depth of flow; $Z_b^{'}$, $Z_b^{''}$ and $Z_b^{'''}$ are the first, second and third derivatives of the bed profile respectively; S_f denotes the friction slope, calculated from the Manning equation or smooth boundary resistance law; Q is the discharge; A is flow cross-sectional area; B is the width of the channel; β refers to the Boussinesq coefficient; g is gravitational acceleration; ρ is the density of the fluid; η is the mean elevation of the free surface; z is the vertical coordinate of a point in the flow field; p is the pressure; and ω_0 is a weighting factor. Fenton (1996) suggested a value of slightly less than one for ω_0. These equations, Eqs. (1) and (2), will be referred to hereafter as the Boussinesq-type momentum equation of uniform centrifugal term (BTMU) model. Following the procedures suggested by Fenton (1996) and assumption of a linear variation of centrifugal term with depth, the following equations are developed for steady flow in a rectangular channel:

$$\frac{Q^2H}{3A}\frac{d^3H}{dx^3}+\frac{Q^2Z_b^{'}}{2A}\frac{d^2H}{dx^2}+\left(gA-\beta\frac{Q^2B}{A^2}\right)\frac{dH}{dx}+gA\left(Z_b^{'}+S_f\right)+\frac{HQ^2}{A}\left(\frac{Z_b^{'''}}{2}+\frac{Z_b^{'}Z_b^{''}}{H}\right)$$
$$-\frac{H^2Q^2}{A^2}\left(\frac{\beta}{H}+Z_b^{''}+\frac{2}{3}\frac{d^2H}{dx^2}\right)\frac{dB}{dx}=0, \tag{3}$$

$$p=\rho\left(g(\eta-z)+\frac{Q^2}{A^2}\left(Z_b^{''}(\eta-z)+\frac{d^2H}{dx^2}\left(\frac{\eta-z}{\eta-Z_b}\right)\left(\frac{1}{2}(\eta+z)-Z_b\right)\right)\right). \tag{4}$$

These equations in this paper are termed the Boussinesq-type momentum equation linear (BTML) model. In the formulation of both models, the curvature at the surface is approximated by $\kappa_H \cong d^2H/dx^2+Z_b^{''}$ and at the bed by $\kappa_b \cong Z_b^{''}$. If Eq. (4) is compared with the corresponding equation, Eq. (2), it is noticed that the term, which accounts for the dynamic effect due to the curvature of the flow surface, shows a quadratic variation in Eq. (4). It should be remarked that Eq. (2) predicts a linearly varying non-hydrostatic pressure distribution. In this work, the two models will be applied to simulate transcritical flows over curved beds with and without lateral contractions. The simulation results of the two models will be compared with measurements to examine the influence of the pressure correction factors on the solutions of the models.

3. PROBLEM FORMULATION AND BOUNDARY CONDITIONS

The computational domain for the numerical solution of the flow problem is shown in Fig. 1. In this figure AB and CD are the inflow and outflow sections respectively, and BD is the curved flow boundary in which the first and second derivatives are continuous. The inflow and outflow sections of the computational domain are located in a region where the flow is assumed to be nearly horizontal, with hydrostatic pressure distribution. This flow condition simplifies the evaluation of the boundary values at these sections using the gradually varied flow equation,

$$S_H = \frac{dH}{dx} = \frac{S_0 - S_f}{1 - \beta Fr^2}, \tag{5}$$

from which for the given depth and discharge at the inflow section, the corresponding slope of the water surface, S_H, can be evaluated numerically. In this equation, Fr is the Froude number and S_0 is the bed slope. For the given boundary values at the inflow and outflow sections, and discharge at the inflow section, it is required to determine the flow surface profile, AC, and the bed pressures along the flow boundary BD. For this purpose, the computational domain of the flow problem is discretised into equal size steps in x as shown in Fig. 1.

4. COMPUTATIONAL MODEL DEVELOPMENT

A numerical solution is necessary since closed-form solutions are not available for these nonlinear differential equations. The finite difference approximations are used to discretise the above flow equations. This formulation is very simple to code and extensively used to solve linear or nonlinear differential equations. For discretisation purpose, Eqs. (1) and (□) can be represented by a simple general equation as

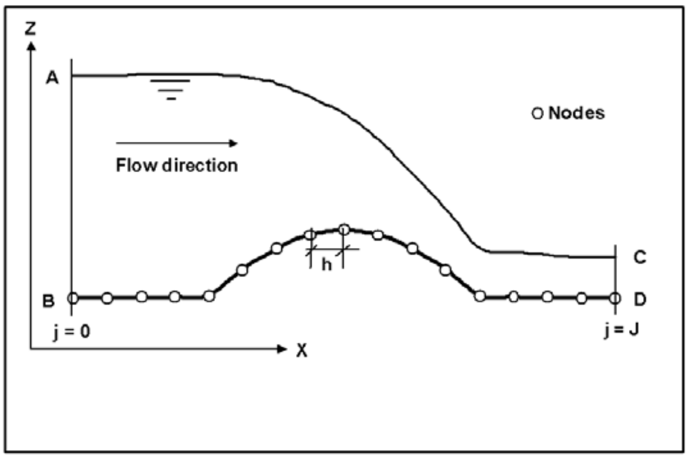

Fig. 1 Computational domain for transcritical flow problem

$$\frac{d^{\square}H}{dx^{\square}} + \xi_0 \frac{d^2H}{dx^2} + \xi_1 \frac{dH}{dx} + \xi_2 + \xi_{\square} = 0, \tag{6}$$

where ξ_0, ξ_1, ξ_2 and $\xi_\square$ are the nonlinear coefficients associated with the equations. Higher-order finite difference approximations are employed here to replace the derivative terms in these equations in order to reduce the truncation errors introduced in the formulation due to the finite difference quotients (see e.g., Fletcher, 1991). The upwind finite difference approximations (Bickley, 1941) for derivatives at node j in terms of the nodal values at $j-2$, $j-1$, j and $j+1$ are introduced into Eq. (6) in places of the derivatives. After simplifying the resulting expression and assembling similar terms together, the equivalent finite difference equation reads as

$$\begin{aligned}&6\xi_{2,j}h^{\square}+H_{j-2}\left(-6+\xi_{1,j}h^2\right)+H_{j-1}\left(18+6\xi_{0,j}h-6\xi_{1,j}h^2\right)\\&+H_j\left(-18-12\xi_{0,j}h+\square\xi_{1,j}h^2\right)+H_{j+1}\left(6+6\xi_{0,j}h+2\xi_{1,j}h^2\right)\\&-\frac{6h^{\square}\mu_1}{B_j}\left(\frac{dB}{dx}\right)\left(\frac{\varphi_j}{H_j}+\frac{\mu_2}{h^2}\left(H_{j-1}-2H_j+H_{j+1}\right)+\mu_\square Z''_{b,j}\right)=0,\end{aligned}\tag{7}$$

where h is the size of the step, and μ_1, μ_2 and $\mu_\square$ are constants related to the model equations and are given in Table 1. Since the value of the nodal point at $j=0$ is known, the value of the imaginary node at $j=-1$ can be determined from the estimated water surface slope, S_H, at the inflow section. Using a similar discretisation equation for the water surface slope at inflow section and the expanded form of Eq. (7) at $j=0$, the explicit expression for the nodal value at $j=-1$ in terms of values of the nodal point 0 and 1 is

$$H_{-1}=\left(\frac{-1}{\Omega+6\Pi}\right)\left(H_1\theta+H_0\Psi+\Pi\left(6hS_H-\square H_0-2H_1\right)+6\xi_{2,0}h^{\square}\right),\tag{8}$$

where:

$\theta=6+6\xi_{0,0}h+2\xi_{1,0}h^2$; $\Psi=-18-12\xi_{0,0}h+\square\xi_{1,0}h^2$; $\Omega=18+6\xi_{0,0}h-6\xi_{1,0}h^2$; $\Pi=-6+\xi_{1,0}h^2$.

Eqs. (7) and (8) constitute a one-dimensional finite difference hydrodynamic model.

The solution of the non-linear flow equation based on the two-point boundary value technique requires a known initial position of the free surface profile. This makes the solution of the flow problems relatively more difficult due to the fact that the location of the free surface profile is not known a priori. Generally, such problems must be solved by iterative methods, which proceed from an assumed initial free surface position. The computational effort for the iteration procedure may be dependent to some extent on the choice of the initial flow surface profile. In this work, the Bernoulli and continuity equations are employed to obtain the initial flow surface profile for commencing the iteration solution. To simulate the flow surface profile, Eq. (7) is applied at different nodal points within the solution domain and this results in a sparse system of non-linear algebraic equations. These equations together with Eq. (8), and the two boundary values at the inflow and outflow sections, are solved by the Newton-Raphson iterative method with a numerical Jacobian matrix. The convergence of the solution is assessed using the following criterion:

$$\sum_{j=1}^{m}\left|\delta H_j\right|\le \text{tolerance},$$

where δH_j is the correction depth to the solution of the nodal point j at any stage in the iteration; m is the total number of nodes in the grid excluding nodes having known values. In this study, a tolerance of 10^{-6} is used for the convergence of the numerical solution. This computational scheme is superior to the method employed by Fenton (1996) which was based on the shooting technique and showed parasitic numerical instability.

For the solution of the pressure equations, a similar finite difference approximation is inserted into Eqs. (2) and (4) to discretise the derivative term in the equations. Since the nodal flow depth values are known from the solution of the flow profile equations, these discretised equations yield the bed pressure (for $z = Z_b$, where Z_b is the channel bed elevation) at different nodal points.

Table 1. Values of the constant parameters for the model equations

Model	μ_1	μ_2	$\mu_□$	φ
BTMU equations	4	1/2	ω_0	$1 + Z_b'^2$
BTML equations	□	2/□	1	1

5. SIMULATION RESULTS AND DISCUSSION

The one-dimensional numerical models presented in the previous section are now used for simulating steady: i) transcritical flow over trapezoidal profile weirs; ii) free flow in venturi flumes; and iii) transcritical flow over a curved bed combined with sidewall curvature. Since the experiments for all test cases were performed in plexiglass laboratory flumes, a smooth boundary resistance law was used to estimate the friction slope for the models. For computational simplicity, β is assumed as unity in both models. All numerical results presented here were independent of the effect of spatial step size.

5.1 FLOW OVER TRAPEZOIDAL PROFILE WEIRS

Experiments on flow over trapezoidal profile weirs were conducted to validate the results of the models at the Michell Laboratory of the University of Melbourne. The flume was 7100 mm long, □80 mm high, and □00 mm wide. The flume and the trapezoidal profile weirs were made of plexiglass. Symmetrical trapezoidal profile weirs of 150 mm high, crest lengths 100 mm, 150 mm and 400 mm respectively and side slope 1V: 2H were tested at different discharges. The flow surface and bed pressure profiles were measured respectively by a point gauge and piezometers. Details of the experimental system can be found in Zerihun and Fenton (2004b).

For assessing the effect of the streamline curvature on the solution of the models, transcritical flow over trapezoidal profile weirs with different H_1 / L_w (H_1 = total energy head over the weir crest, L_w = weir crest length) values were considered. The computed flow surface and bed pressure profiles for this flow situation are compared with experimental results in Figs. 2 and □ The BTMU and BTML models solutions for flow surface profile show excellent agreement with the measured data. Both models predict similar flow surface profiles and steep flow surface slope on the crest of a short-crested weir. A minor discrepancy between the results of the two models for bed pressure profiles can be seen from these figures.

In general, the overall qualities of the numerical solutions of the bed pressure are good and show good agreement with the experimental data.

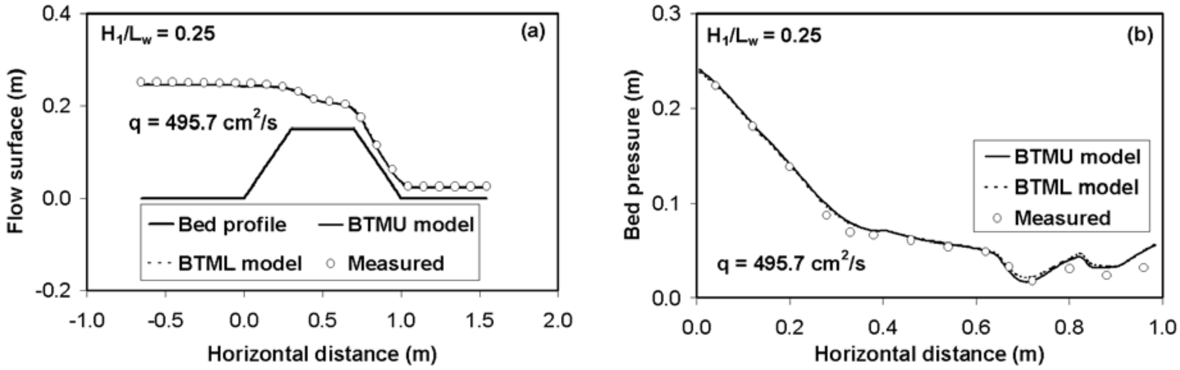

Fig. 2 Flow surface and bed pressure profiles for flow over a broad-crested weir

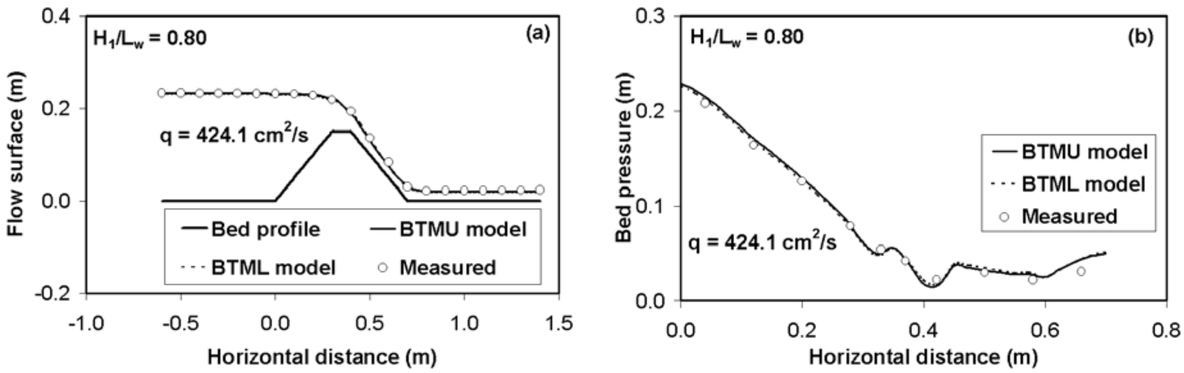

Fig. □Flow surface and bed pressure profiles for flow over a short-crested weir

5.2 FREE FLOW IN VENTURI FLUMES

5.2.1 Ye and McCorquodale's (1997) Experiment

Ye and McCorquodale (1997) carried out an experiment in a Parshall flume model at the University of Windsor. This flume consists of three sections: a converging inlet section with a horizontal bed and variable widths to create the critical depth; a throat section with parallel sidewalls and a sloping bed in which supercritical flow occurs; and a diverging outlet section with an adverse sloping bed and variable widths. Result of the flow surface profile measurement of this test was used for the verification of the model solutions.

Fig. 4 shows the computational results of the two flow models for transcritical flow in a Parshall flume. This figure compares the cross-channel averaged experimental and numerically predicted flow surface profiles along the centreline of the flume. The two models accurately simulate the flow transition from sub- to super-critical state for this flow condition. For the upstream flow region ($x \le 0.8\,\mathrm{m}$), the agreement between the experimental and numerical results is good, and no significance differences are observed between the computational results of the two models for the entire flow region. However, a minor discrepancy between the models and experimental results can be seen from this figure in the supercritical flow region downstream of the diverging section of the flume ($x > 0.8\,\mathrm{m}$). In this flow region, the models slightly underestimate the flow surface elevation. As a one-dimensional model, these models do not describe the existence of cross-waves downstream of the end of the throat section.

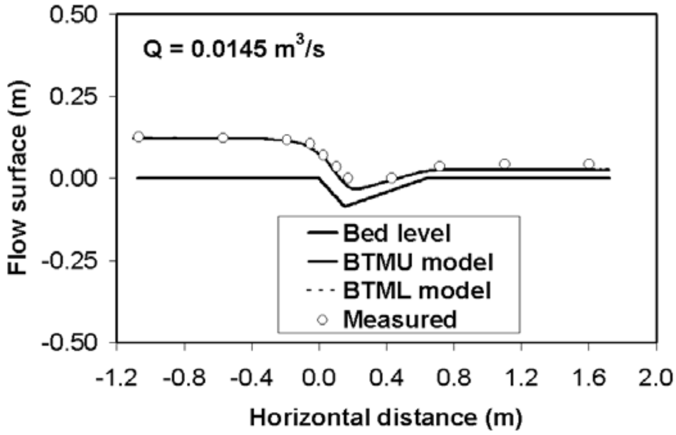

Fig. 4 Comparison of computational and experimental results for flow in a Parshall flume

5.2.2 Khafagi's (1942) Experiment

The experimental results of Khafagi (1942) are invoked to test the ability of the model to simulate transcritical flow in a venturi flume. The test flume with horizontal bed consists of three sections: a converging inlet section with a rounded constriction (Radius = 545 mm) to create critical depth; a short throat section with parallel sidewalls; and a diverging outlet section (side slope 1:8) with a gradually varied width. The uncontracted width of the flume was □00 mm. A series of experiments were conducted using this flume. The flow surface and pressure profiles measurements of these tests were used to validate the models. Fig. 5 shows comparisons between measured and computed flow surface profiles along the centreline of the flume for transcritical flow with supercritical outflow condition downstream of the throat section. It can be seen from this figure that the two models simulate similar flow surface profiles for the entire flow region which agree very well with experimental data. In both cases, the flow transition is accurately simulated.

In order to examine the influence of the applied corrections for the dynamic pressure in the local flow characteristics, the pressure distributions at the end of the converging ($x =$ □0 cm) and at the beginning of the diverging ($x =$ □5 cm) sections of the flume are simulated using these models. Fig. 6 shows the comparison of the experimental data and the numerical predictions of these models for the pressure distributions. In this figure the non-dimensional pressure distribution at a section, p/p_0 (p = pressure at height, h_s , above the bed, $p_0 = \rho g H$) is shown versus the non-dimensional height above the bed, h_s/H . In both cases of pressure distribution simulations, the results of the BTML model show a slightly better agreement with measurements than the predictions of the BTMU model.

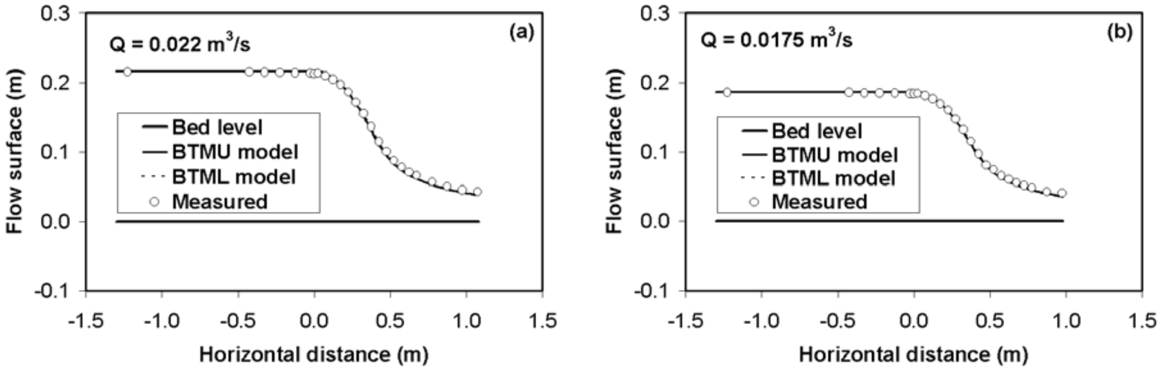

Fig. 5 Comparison of predicted and measured flow surface profiles for free flow situation

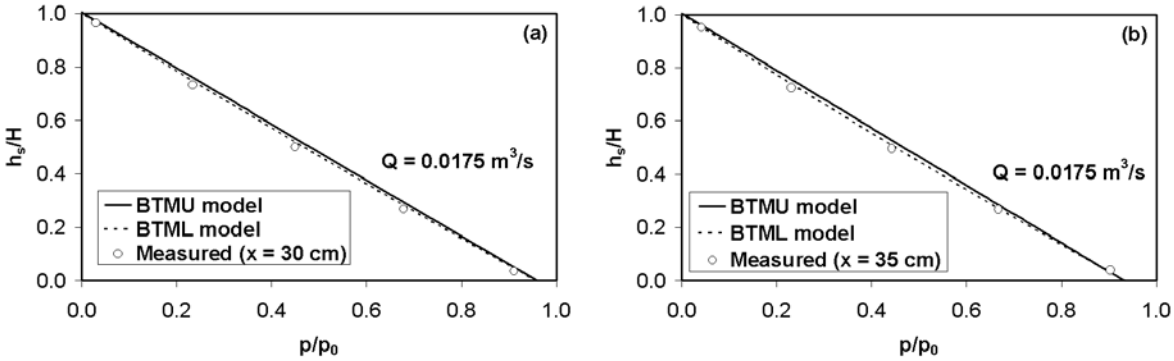

Fig. 6 Comparison of computational and experimental results for pressure distribution

5.3 FLOW IN A CHANNEL WITH CURVED BED AND SIDEWALL

The results of the experiments conducted by Law (1985) are selected to test the predictions of the models. A 16 m long recirculating glass wall flume was used to conduct the experiment. An obstacle made of plexiglass was installed at the middle of the flume to bring on simultaneous geometric changes in the bottom and sidewall of the flume. Due to this obstacle, the width of the channel was reduced to a minimum of 94 mm and the bed was elevated to a maximum height of 6□ mm. The staggered distance between the maximum contraction and the maximum height of the hump was 200 mm. A point gauge of reading accuracy 0.□0 mm was used to observe the flow profiles at the middle of the cross-section. Further details of the experimental set-up and procedures are given in Law (1985).

A numerical simulation result based on the shooting method was reported by Law (1997) for this flow problem using the Fenton's equation. Similar to the Fenton (1996) solution, however, Law’s simulation result suffered from strong parasitic numerical instability besides the effects of bed profile discontinuity. In this work, smooth transition curves at both ends of the hump were introduced for smoothing the existing discontinuities.

Fig. 7 shows the comparison of the experimental data with the numerical predictions of the models for low and high flow cases. In the flow regions upstream of the minimum width of the flume ($x < 0$) and downstream of the maximum height of the hump, the BTML and BTMU models simulate similar flow profiles for both flow cases and are in good agreement with measurements. However, some differences are observed between the solutions of these models in the transition region where the combined effects of the curvatures of the bed and sidewalls are substantial. In this region ($0 \le x \le 0.20\,\mathrm{m}$), the performance of the BTML model is marginally better than the BTMU model. As can be seen from this figure, the predictions of these models slightly depart from measurements for high flow condition. This is probably due to the influence of the horizontal curvature of the streamline which is more significant in the transition region especially at higher discharge.

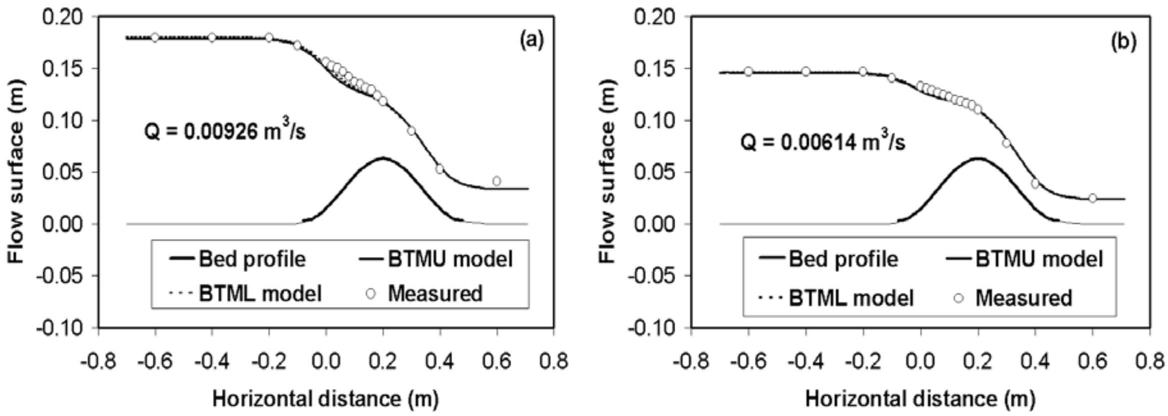

Fig. 7 Comparison of predicted and measured flow profiles for transcritical flow situation

As already indicated, the assumed distribution shapes of the centrifugal term determine the degrees of the resulting corrections for the effect of the non-hydrostatic pressure distribution. The overall simulation results of this study demonstrate that the proposed approximations for these corrections have only a marginal effect on the predictions of flow surface and bed pressure profiles. However, the pressure distribution simulation results of the considered flow problems are very sensitive to these approximations. Zerihun and Fenton (2004a) pointed out that the influence of such approximations for the effects of the centrifugal pressure is more significant in the prediction of local flow characteristics of a curved flow than the global flow characteristics.

6. SUMMARY AND CONCLUSIONS

Two one-dimensional Boussinesq-type momentum equation models were investigated for simulating transcritical flow over curved beds with and without the effect of lateral contractions. Uniform and linear distribution shapes for the variation of the centrifugal term were used to develop the equations. Finite difference approximations were employed to discretise the flow equations. The Newton-Raphson iterative method with a numerical Jacobian matrix was applied for the solutions of the resulting nonlinear algebraic equations. Comparison of the numerical prediction results with experimental data was also presented.

A good agreement was observed between the predicted and measured values. Results of this investigation reveal that the 2-D flow structures for the transcritical flow situation are better described by the proposed model which incorporates a higher-order correction for the effect of the centrifugal pressure. The results also demonstrate the superior performance of the current computational scheme compared to a previous scheme based on the shooting method for the solutions of such equations. This study suggests that a higher-order pressure equation should be used when accurate simulation of the pressure distribution of a flow with pronounced curvatures of streamlines is sought.

REFERENCES

Bickley, W. G. (1941). *Formulae for Numerical Differentiation*, Math. Gaz. Vol. 25, pp. 19-27.

Bos, M. G. (1978). *Discharge Measurement Structures*, Wageningen: ILRI.

Boussinesq, J. (1877). *Essai Sur la Théorie des Eaux Courantes*, Mémories Présentés par Divers Savants à l'Académie des Sciences, Paris, Vol. 2□, No.1, pp.1-680.

Dressler, R. F. (1978). "New nonlinear shallow flow equations with curvature", *J. Hydr. Res.*, Vol. 16, No.□, pp. 205-220.

Fenton, J. D. (1996). "Channel flow over curved boundaries and a new hydraulic theory", *Proc. 10th Congress, APD-IAHR,* Langkawi, Malaysia, Vol. 2, pp. 266-27□

Fletcher, C. A. J. (1991). *Computational Techniques for Fluid Dynamics 1*, Springer-Verlag Berlin Heidelberg, Germany.

Hager, W. H., and Hutter, K. (1984). "Approximate treatment of plane channel flow", *Acta Mechanica,* Vol. 51, pp. □1-48.

Hager, W. H. (1985). "Equation of plane, moderately curved open channel flows", *J. Hydr. Engrg., ASCE,* Vol. 111, No. □, pp. 541-546.

Khafagi, A. (1942). *Der Venturikanal: Theorie und Anwendung*, Eidgenössiche Technishe Hochschule Zürich, Mitteilungen der Versuchsanstalt für Wasserbau und Erdbau, No. 1, Zürich.

Khan, A. A., and Steffler, P. M. (1996a). "Vertically averaged and moment equations model for flow over curved beds", *J. Hydr. Engrg., ASCE,* Vol. 122, No. 1, pp. □-9.

Khan, A. A., and Steffler, P. M. (1996b). "Modelling overfalls using vertically averaged and moment equations", *J. Hydr. Engrg., ASCE,* Vol. 122, No. 7, pp. □97-402.

Law, A. W. K. (1985). *Single Layer Flow Over an Obstacle Consisting of a Contraction and Sill*, Hydraulic Engineering Laboratory Report No. UCB/HEL-85/06, Department of Civil Engineering, University of California at Berkeley.

Law, A. W. K. (1997). "Steady flow over an obstacle with contraction and sill", *Proc. 27th Congress, IAHR*, San Francisco, pp. 787-792.

Matthew, G. D. (1991). "Higher order, one-dimensional equations of potential flow in open channels", *Proc. Instn. Civ. Engrs.*, London, England, Vol. 91, pp. 187-201.

Steffler, P. M., and Jin, Y. (199□). "Depth averaged and moment equations for moderately shallow free surface flow", *J. Hydr. Res.*, Vol. □1, No. 1, pp. 5-17.

Ye, J., and McCorquodale, J. A. (1997). "Depth-averaged hydraulic model in curvilinear collocated grid", *J. Hydr. Engrg., ASCE*, Vol. 12□, No. 5, pp. □80-□88.

Zerihun, Y. T., and Fenton, J. D. (2004a). "Boussinesq-type momentum equations solutions for steady rapidly varied flows", *Proc. 6th International Conference on Hydro-science and Engineering,* Brisbane, May □1-June □, CD-ROM.

Zerihun, Y. T., and Fenton, J. D. (2004b). "A One-dimensional flow model for flow over trapezoidal profile weirs", *Proc. 6th International Conference on Hydro-science and Engineering*, Brisbane, May □1-June □, CD-ROM.