A model investigation of squat

by B. B. Sharp and J. D. Fenton

In the course of conducting a model investigation of the surge effects of shipping in a restricted water way and an adjacent dock, some observations of squat were made (ref. 1).

In the investigation, a 1/150th scale model of the m.v. Jordaens (ref. 2) was used. The dynamic characteristics were carefully reproduced so that heave, pitch and surge could be studied. It is of interest to report the model results because of the small scale that was used.

Additionally, momentum methods have been applied for comparison with the customary energy model (ref. 3 and 4) and other published results of squat observations.

Three approaches might be used to analyse the phenomenon of a ship squatting in a restricted water way. The simplest interpretation is undoubtedly found in the Energy method where it may be appreciated that the blockage of a ship as it moves in a waterway produces a kinetic head \(\frac{v^2}{2g}\) in the water around the ship. To conserve energy (Bernoulli principle) there must be a decrease in pressure head (water level falls) and because of buoyancy the ship squats.

The extent of squat can be readily expressed by this means as a function of the ship’s Froude Number relative to the water ahead and its blockage factor. There also appears to be a “critical” condition when the ship rides up with reduced average squat (ref. 3).

A fundamental hydrodynamic approach (ref. 5 & 6) has been developed and would appear to be of greatest value in the refinement of allowing for detailed ship geometry. Vertical forces, moments and wave resistance may be evaluated by this method.

Thirly, the momentum principle may be applied. The power of this method lies in the ability to make a judicious choice of the control volume so as to reduce the effect of unknown forces. Additionally, some aspects of ship geometry and channel shape may be taken into consideration which may not be, by the energy method.

The momentum principle (see fig. 1) requires a statement of the forces \(F\) and moment \(M\) as follows:

To reduce the problem to steady flow, the velocity \(V\) of the ship is superimposed and then,

\[
F_1 = \frac{w}{2} B y_1^2
\]

\[
F_2 = \frac{w}{2} \frac{A}{h} (y_1 - y_s + h)^2
\]

\[
F_3 = \frac{w}{2} (B y_1^2 - Ah)
\]

\[
M_1 = -B y_1 V^2
\]

\[
M_2 = \frac{w}{g} (B y_s^2 - A) (V + v)^2
\]

In the above, the symbols are defined in figure 1 and \(w\) is the specific weight of water, \(g\) the acceleration due to gravity and \(K_1\) is a coefficient in the force allowing for stagnation pressures.

The continuity equation for the steady flow case is

\[
(V + v) (By_s - A) = V y_1 B
\]

Equating force plus momentum yields

\[
F^2 = \frac{(1-d-S)}{(1+S)} \left[ 1 - \frac{2}{d} \left( \frac{p}{K_1} + \frac{1}{S} - (1 - K_1) - S K_1 \right) \frac{d}{Sp} \right]
\]

where \(F\) is the Froude Number \(V/(gy_s)^{\frac{1}{2}}\) and the dimensionless variables are

\[
d = (y_1 - y_s) / y_1 = \text{squat}
\]

\[
S = A / By_s = \text{blockage factor}
\]

\[
p = h / y_1 = \text{relative draught}
\]

For comparison the energy method yields the relationship

\[
F^2 = \frac{S}{d} \left( \frac{1}{(1+S)} - S \left( \frac{d}{2d} + \frac{d}{2s} \right) \right)
\]

Structurally the most significant difference is in the denominator in equation (8) where the additional term shows that \(F\) using the energy method will be greater than for the momentum theory. Conversely for a given \(F\), the momentum theory suggests greater squat.

The two methods are compared in figure 2 for the same \(S\)
value and assumed $p$ values, for the case of a rectangular waterway and $K_1 = 1$. (Values of $K_1$ greater than 1 cause greater squat.)

The above presentation of momentum is, however, a simplification of the problem, particularly when model experimental results are being considered. Constantine, for example (reference 3), states that pile up due to a friction bore will be greater in a model than in the prototype and will cause less flow past the ship.

In fact, when a long reach is considered, the pile up for a fixed $V$ becomes constant, the problem is again rendered a steady one by the addition of $V$ and the only change in the equations will be an additional force.

$$ F_s = -K_2 (V + v)^2 (By_3 - A) $$

(9)

The same continuity equation applies although the value of $v$ will now be different at the ship due to the change in momentum associated with the addition of $F_s$.

The value of $F_s$ will be dependent on the frictional drag force, on the hull shape and in general will be greater in the model than in the prototype.

Equation (7) is then modified in one term only, thus:

$$ F^3 = \frac{(1-d-s)}{s} \left[ \frac{d}{2} \frac{K_1}{p} (1 - S) + \frac{s}{d} \frac{Sp}{d} \right] $$

(10)

It will be difficult to estimate $K_2$ analytically, since it is a combination of wave drag, pressure drag and friction drag. To assess its importance, therefore, the value of $K_2$ to produce agreement between equation (10) with $K_1 = 1$ and the experimental results will be calculated (see later).

It should be noted that a significant value of $K_1$ greater than unity is not implied because of "pile-up", since this takes place a little distance ahead of the vessel without necessarily causing a greater wetted area immediately at the bow.

Experimental Facilities

The model scale was dictated by the availability of a 6 ft. wide towing tank 120 ft. long with a self propelled Kempf and Remmer design towing carriage and electronic control of speed.

The Jordauens was supported by a yoke pivoted at its pitch axis and the whole was supported by a cylinder floating in mercury.
The additional buoyancy of the support system amounted to 22 per cent of the Jordans' own buoyancy and so the experimental data has been multiplied by a factor of 1.22. (It can be shown analytically that this amount of restraint has a very small affect and can be allowed for by this linear factor.)

The heave (squat) was measured by a small cantilever beam with strain gauges forming part of a Wheatstone Bridge circuit. Similarly the pitch was measured by a small cantilever attached to the yoke and deflected by the relative pitching of the ship with respect to the yoke. The heave and pitch were recorded simultaneously on an ultra violet galvanometer recorder.

A range of speeds was tested with the water depth at approximately 2.9 in. and tank widths of 6 ft. and 3 ft.

Experimental Results
The tests corresponded to the following conditions:

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<tr>
<th>Test</th>
<th>B</th>
<th>h</th>
<th>A</th>
<th>S</th>
<th>p</th>
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The results have been plotted directly on F versus d graphs in Figures 3 and 4. The momentum equation (7) has been incorporated as well.

The experimental results include both the mean overall ship squat as well as the greater stern squat determined from the pitch measurements.

The critical squat condition agrees quite well with the theoretical value (mean squat curves).

The theoretical curves exclude the drag effects included in equation (10). The effect of neglect of drag is shown by evaluating $K_2$ and this is illustrated graphically in Figure 5.

It is clear from these results that the "drag coefficient" $K_2$ behaves as one would expect, decreasing as the velocity (F) increases. The values are quite high, though they represent the cumulative effect of all the recognised drag components. The squat is obviously a very sensitive indicator of these effects.

Finally, the stern squat is influenced by the drag and in the prototype would also be less. The result agrees quite well in form for large F values with that published by Constantine for tests on a 1/144 scale model.

Conclusions
(1) Model tests may be used to predict squat provided some allowance is made for the greater drag effects occurring in the model.

(2) The momentum theory may be usefully applied to estimate squat and has the advantage of incorporating terms to allow for relative draught, channel cross section and drag forces.

Acknowledgment
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REFERENCES