Workshop on Modernization of Irrigation Water Delivery Systems, US Committee on Irrigation and Drainage, Phoenix, Arizona; October 1999

On the nature of waves in canals and gate stroking and control

John D. Fenton[1], Anthony M. Oakes[2] and David J. Aughton[2]

[1] Professorial Associate, Department of Civil and Environmental Engineering, The University of Melbourne, Parkville, Victoria, Australia 3052

[2] Software Applications Manager and Managing Director respectively , Rubicon Systems Australia, P.O. Box 114, Camberwell, Victoria, Australia, 3124.

Abstract

The long wave equations governing wave motion in canals are considered, and a low Froude number approximation introduced. This reduces the problem of computing wave motion in canal systems to solving a single differential equation, and should allow rather simpler programs and larger time steps. The problem of computing upstream gate motions to bring about desired downstream flows at a regulator is then considered. Theory suggests, and computational results show, that the very idea of computing all the details of the upstream gate motions is flawed. It is more reasonable just to use the upstream gate motions to satisfy approximately the downstream flow requirements and to implement control measures at the downstream regulator.

Introduction

A common misconception concerning the nature of wave motion in waterways is that all disturbances travel at the so-called long wave speed. In fact, for rivers that are relatively rough and steep, disturbances travel without much diminution at the kinematic wave speed, which is about 1.5 times the flow velocity. For typical conditions in irrigation canals which are relatively smooth and with mild slopes, the underlying speed of propagation is also the kinematic wave speed, but where there is also an apparent diffusion, such that disturbances tend to diminish in height and to spread out in space and time. The effect is such that outflow hydrographs from a pool are quite different from the inflow hydrograph supplied to the pool, which has a number of implications for computing gate stroking.

The concept of gate stroking was introduced by Wylie (1969), with the aim of calculating the movements of an upstream gate in a canal so as to bring about a desired variation in discharge at a downstream gate with the aim of minimising surface disturbances. In more recent years the concept has often been described more as one of “feedforward” control, where control measures are applied in anticipation of an expected event in order, for example, to avoid unacceptable changes in downstream water levels. The movements of the upstream gate are designed to supply as much as possible the desired flow history at the downstream gate with a minimum of operation of the downstream gate and subsequent wave generation.

There have been many papers devoted to the subject since 1969, and although much has been claimed for the variety of methods developed, many of the results obtained have been unsatisfactory. Gate stroking as such seems to have been applied little in practice, although the American Society of Civil Engineers Task Committee on Canal Automation Algorithms has given some emphasis to feedforward control (see the series of papers introduced by Clemmens, Burt, and Rogers, 1995).

As canal systems are progressively moving to real-time control, the opportunity now exists to take advantage of modelling and system identification techniques employed by process control industries. Rubicon Systems Australia is developing such control systems that are based on observed data and are less computationally demanding than physical models. This development is using "grey box" system identification where prior physical knowledge is incorporated into physical models. This approach makes use of the best features of physical modelling and system identification in order to achieve near-optimal canal control. As demand forecasting systems also become integrated with control systems, canal operations are not only taking advantage of pre-emptive distant upstream or downstream actions, but can also rely on future control strategies based on demand predictions. Rubicon is also extending the technique of system identification into the area of demand prediction as an integrated canal control technology.

In this paper, we examine the equations governing the motion of waves and flows in canals. We develop a low-Froude number approximation that should allow simpler and faster programs for the simulation of waves in canal systems. We then show that the technique for gate stroking, of stepping backwards in space and time, is computationally defective. It is equivalent to computing with negative diffusion such that a downstream change that is too sudden will require huge and unreasonable upstream gate motions. We present a means of computing the problem, which can make use of existing software without any modification to solve this reverse problem. However, in general we warn against the detailed computation of gate stroking and suggest that more heuristic measures are justified, including the aid of established practice, the use of accurate numerical simulation, and most importantly, control measures at the downstream gate.

Theory for waves in canals

The flow of water and the propagation of waves in canals are described well by the long wave equations. Here we present them in the form where the dependent variables are the cross-sectional area *A* and discharge *Q*:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q, \tag{1}$$

$$\frac{\partial Q}{\partial t}+\left(\frac{gA}{B}-\beta\frac{Q^2}{A^2}\right)\frac{\partial A}{\partial x}+2\beta\frac{Q}{A}\frac{\partial Q}{\partial x}=gA\left(\overline{S}-S_f\right)+qu_q-\frac{Q^2}{A}\frac{d\beta}{dx}, \tag{2}$$

where x is the distance along the waterway, t is time, q is inflow per unit length, which initially has a velocity u_q, g is gravitational acceleration, B is the width of the surface, β is the Boussinesq momentum coefficient, S_f is the friction slope, and $\overline{S}$ is the mean bed slope at a section. The equations in this form have been obtained by Fenton (1999). Usually S_f is approximated by a friction law such as Manning's or Chézy's law, which we will generalise here such that we write $S_f = Q^2/K^2$, where K is the conveyance, which can be expressed in terms of the roughness, area A, and wetted perimeter P of the cross-section. It is possible to recast these two partial differential equations as four ordinary differential equations. In the so-called Characteristic Formulation, it is possible to deduce that information (in the form of the gradient of the characteristics) proceeds up and down the canal at a speed given by $\sqrt{gA/B}$, which can be interpreted as $\sqrt{g\times \text{Mean depth}}$. A common interpretation is that all disturbances actually travel up and down the canal at this speed. While this is true if there is no friction, in general it is not correct as friction changes the behaviour of the waves considerably. In fact, for typical irrigation canals, waves may be markedly diffused so that they arrive downstream considerably diminished in height and spread out much more in space and time. Insight into this process can be had if we consider approximations to equations (1) and (2), as we now set out to do.

The small Froude number approximation

Equations (1) and (2) can be non-dimensionalised, see, for example, Strelkoff and Clemmens (1998). The possibly surprising result is obtained that not only are the momentum flux terms involving β of order of the square of the Froude number, but so is the $\partial Q / \partial t$ term. If the Froude number is sufficiently small, the original dimensional momentum equation (2) can then be approximated by:

$$\frac{1}{B}\frac{\partial A}{\partial x} + S_f - \overline{S} = 0 \,, \tag{3}$$

ignoring usually unimportant and inflow terms on the right side. The neglected terms are of the order of the square of the Froude number, which is small in most irrigation canals. This level of approximation is the basis of much work in flood studies, which leads to the kinematic wave approximation and advection-diffusion routing (see, for example, Singh, 1996). There is a possibility that rapid gate movements in irrigation canals might impose rapid flow changes ($\partial Q / \partial t$ large) on the canal, but the effects of this will have to be tested in practice.

We now substitute the generic friction law $S_f = Q^2 / K^2$, so that equation (3) becomes

$$\frac{1}{B(A)}\frac{\partial A}{\partial x} + \frac{Q^2}{K^2(A)} - \overline{S} = 0 \,, \tag{4}$$

where we have shown both the breadth *B* and the conveyance *K* as functions of the area *A*. This is a considerably simpler dynamic equation than is (2). We now show that it can be used as the basis for approximately modelling disturbances in irrigation canals.

Volume potential and a volume routing equation

The mass conservation equation (1) is linear, and is in such a form that it suggests the introduction of a canal potential *V* such that

$$A = \frac{\partial V}{\partial x} \text{ and } Q = -\frac{\partial V}{\partial t} + \int_{x_0}^{x} q(x')\, dx' \,, \tag{5}$$

where x_0 is arbitrary, typically at the upstream end of the region of interest. If *V* satisfies these equations then the mass conservation equation (1) is identically satisfied. In many situations, particularly in irrigation, there will be no inflow and then *V* simply satisfies $Q = -\partial V / \partial t$. In fact $V(x,t)$ is simply the volume upstream of point *x* at time *t*. The derivative of volume with respect to *x* gives the cross-sectional area, and as $\partial V / \partial t = \int_{x_0}^{x} q(x')\, dx' - Q$, the time rate of change of *V* at a point is the total rate upstream at which the volume is increasing, which is $\int_{x_0}^{x} q(x')\, dx'$ less *Q*, the volume rate which is passing that point.

If we substitute the volume potential formulation into the simplified momentum equation (4), it becomes a partial differential equation in the single variable $V(x,t)$:

$$\frac{1}{B(V_x)}\frac{\partial^2 V}{\partial x^2} + \frac{\left(\int_{x_0}^{x} q(x')\, dx' - V_t\right)^2}{K^2(V_x)} - \overline{S} = 0 \,, \tag{6}$$

in which subscripts denote partial differentiation. Re-arranging this, we obtain an equation that we term the *Volume Routing Equation*:

$$\frac{\partial V}{\partial t} + K(V_x)\sqrt{\overline{S} - \frac{1}{B(V_x)}\frac{\partial^2 V}{\partial x^2}} = \int_{x_0}^{x} q(x')\,dx' , \tag{7}$$

where we have chosen the sign of the square root such that $\partial V / \partial t$ will be negative, corresponding to Q being positive. This should describe the propagation of disturbances in waterways at low Froude number. Usually the inflow term on the right hand side will be zero, and for the rest of this discussion we will ignore it. If necessary, it can be easily reintroduced.

We examine the nature of the equation by considering small perturbations about a steady uniform flow of area A_0 and discharge Q_0 such that we write $V = A_0 x - Q_0 t + \varepsilon\psi(x,t)$, where ε is a small quantity which expresses the magnitude of perturbations about the base flow. We substitute into equation (7) and expand as a power series. At first order we obtain

$$\frac{\partial \psi}{\partial t} + c_0 \frac{\partial \psi}{\partial x} = \frac{K_0}{2\sqrt{\overline{S}}\,B_0}\frac{\partial^2 \psi}{\partial x^2} , \tag{8}$$

where $c_0 = K_0^{'}\sqrt{\overline{S}}$ is an advection velocity, in which $K_0^{'} = dK/dA$ evaluated at A_0. This has shown that, for small disturbances, equation (7) becomes the linear advection-diffusion equation, used in flood routing. For Manning's law in *S.I.* units, $K = 1/n \times A^{5/3}/P^{2/3}$, and the advection velocity is

$$c_0 = U_0\left(\frac{5}{3} - \frac{2}{3}\frac{A_0}{P_0}\frac{dP_0}{dA_0}\right), \tag{9}$$

where P_0 is the wetted perimeter corresponding to the reference flow of area A_0. This advection velocity is roughly 1.5 times the mean fluid speed in the canal. Importantly, the waves also show diffusive behaviour, as shown by the second derivative on the right of equation (8), with a diffusion coefficient $K_0 / 2\sqrt{\overline{S}}\,B_0$.

This suggests that in typical rivers and irrigation canals, the bulk of disturbances do not travel as waves of translation at a speed of $\sqrt{g \times \text{Depth}}$ with little change. Rather they are carried at a speed of about 1.5 times the flow velocity and are subject to diffusion such that the wave that arrives at the downstream end is lower and longer than that which entered upstream.

There is a possibly surprising result contained here, however. For waterways which are rougher (conveyance K_0 smaller) and steeper (slope $\overline{S}$ larger), such as rivers in steeper country, the diffusion coefficient on the right of equation (8) is smaller, and so are the effects of diffusion. In the limit of a rough steep river, the whole motion is that of a kinematic wave which travels with little diminution, where the effects of gravity and friction are dominant and balanced. In irrigation canals, which are smoother and with shallower slopes, the diffusion coefficient is larger, such that the motion is dominated by the effects of diffusion rather than the simple translation of the wave.

The volume routing formulation seems to be able to incorporate boundary conditions satisfactorily. At the upstream end of the computational domain, usually the inflow is specified as some function of time, which we might write as $Q(x_0,t) = Q_0(t)$, and the second equation of (5) at $x = x_0$ gives the ordinary differential equation $dV(x_0,t)/dt = -Q_0(t)$, which can be solved analytically or numerically using standard means to give a representation for $V_0(t)$. At check gates an equation connects the flow with the surface elevation on one or both sides of the gate, which we write as $Q = f(\eta_1, \eta_2)$, where

$\eta_{1,2}$ are the water surface heights upstream and downstream of the gate and $f()$ denotes a functional relationship, which can often be written in terms of the head difference: $Q = f(\eta_1 - \eta_2)$. From the cross-sectional information we can express the surface elevation at a point as a function of the cross-sectional area, and as the area can be expressed in terms of $\partial V / \partial x$ from equation (5), we can generally write $\partial V / \partial t = F(\partial V / \partial x|_1, \partial V / \partial x|_2)$ for the check gate, where $F()$ denotes another functional relationship. If we know the variation with x on either side of the gate, we can evaluate $\partial V / \partial t$ at the gate and obtain numerical values for V as a boundary condition as part of the time-stepping procedure.

Here we have shown that the single nonlinear equation (7) is a small Froude number approximation, and it should be able to describe transients in many irrigation canals, with the possible exception of where discharge changes are imposed quickly. Numerical solution might be rather simpler than existing schemes for solving the full equations, and it might be useful in practice in the numerical simulation of canal systems. For example, the underlying velocity is the advection velocity, which is about $1.5U$, whereas in the full equations the magnitude of the velocities with which information propagates up and down the canal is the usually larger dynamic wave speed, giving rise to some computational difficulties.

There are a number of computational models for the full equations that are satisfactory for single canals. However, the properties of the full equations can make the methods rather complicated, where different components of the canal have different lengths, and where the system is topologically more complicated, with branching canals. However, the volume routing equation is an advection-diffusion equation, and such equations can prove demanding to solve. It will be necessary to test the equation in a number of applications and develop fast numerical methods. It is currently under development for an irrigation area in south-eastern Australia.

An example – wave propagation in a pool

Here we examine the behavior of solutions of the volume routing equation and demonstrate the nature of wave propagation in a pool. As a basis for comparison we use the program developed by Rubicon Systems Australia (Rubicon, 1998), which solves the full equations (1) and (2) using a specified-interval characteristics method with spatial approximation by cubic splines and a time-stepping scheme with Richardson extrapolation to gain high accuracy.

We consider as a test case the first pool of Example Canal 2 of Clemmens *et al.* (1998). The pool is 7km long, bottom width of 7m, batter slopes of 1.5:1, a longitudinal slope of 0.0001, a target depth of 2.1m at the check gate, and Manning's $n = 0.02$. To perform the simulation we adopted the general conditions of their Test 2-1, with an initial flow of $10\ \mathrm{m}^3/\mathrm{s}$. The inflow was increased by 25% in 15 minutes. We developed a program to solve the volume routing equation (7) using similar approximation methods to the full model, with cubic spline approximation along the canal and simple Euler forward time stepping but with Richardson extrapolation. We embedded it in the full model. With this naïve Euler method for advancing the solution in time, we found that the new method required even smaller time steps for stability than did the full model. This is currently being worked on and some better methods including upwinding are being developed. The two programs simulated conditions in the canal for several hours, with an overshot weir at the check, as we only wanted to include one pool in our computations at this stage.

To demonstrate the behaviour of the canal initially the flow from the headworks was increased uniformly over 15 minutes and a constant downstream gate opening was maintained. After the flow and surface level started to increase downstream, the check gate was brought up to the required full flow in 15 mins under idealised control, and thereafter required delivering precisely the required increased flow. Figure 1 shows the resulting hydrographs. It can be seen that in this canal with a relatively mild slope and moderate friction that the outflow hydrograph is very different from the inflow hydrograph. The effects of the diffusion-like term with the second derivative in the full nonlinear equation (7) are strong. The time when the dynamic model first showed some effect downstream (about 1/2 hour) corresponded closely to the calculated travel time of a dynamic wave, showing that the forerunner of the motion was a dynamic wave. However, the bulk of the motion is a relatively slow-moving kinematic-diffusion wave, which the approximate model closely predicts. Until the downstream gate was opened at 1.5 hours, only about half of the increased flow had arrived. Calculations based on the kinematic wave speed showed an expected travel time of about 2.25 hours, and from the figure it is clear that this is a more representative travel time for the whole increase of flow. What is also obvious, of course, is how efficacious the opening of the downstream gate was in bringing the flow up to required levels. Relying on the slow movement of the flow transients is not enough.

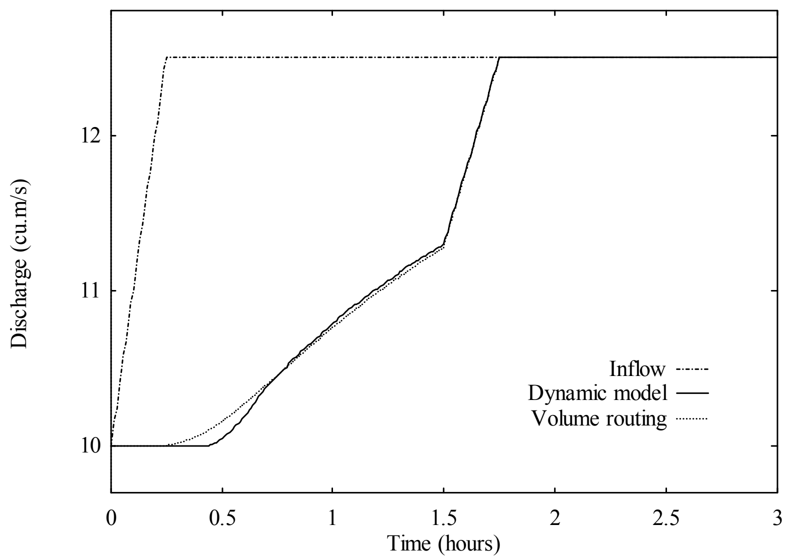

Figure 1. Hydrographs showing inflow and the outflow calculated from the full equations (1) and (2) and from the volume flow routing equation (7).

In our simulation, as described, we suddenly opened the gate and thereafter maintained the desired flow. Under such conditions, what might now be a concern is the behaviour of the water level at the gate. This is shown in Figure 2. It can be seen that there is an initial period, corresponding to a time of rapid changes in the flow, when there was a noticeable disagreement between the two models. However, the approximate model did describe well the main feature of the flow, the increase of flow and surface height as the slow-moving kinematic wave approached. After the gate opened suddenly, as one would expect, the level was quickly drawn down, and again this is described by the approximate model. The volume routing method seems to be capable of simulating the behaviour of the pool, with some errors where rapid flow changes occur, but the overall behaviour of the movement of water masses and surface level behaviour are described satisfactorily.

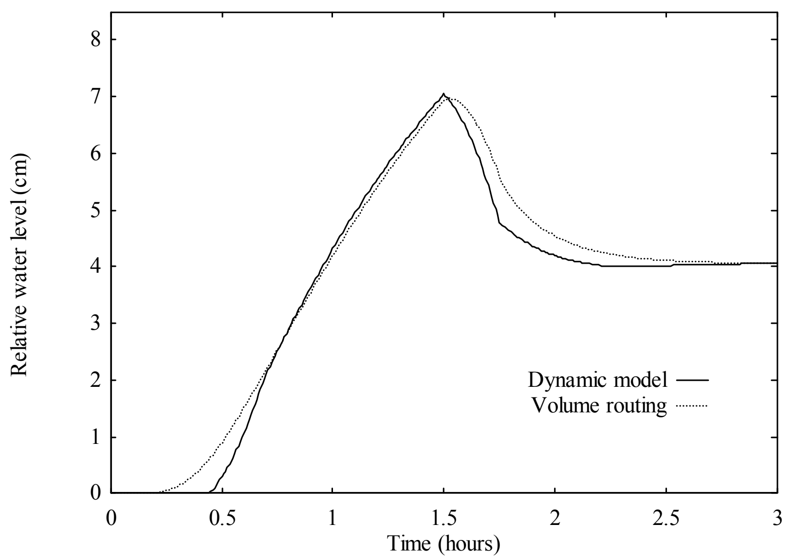

Figure 2. Variation of water level at the gate calculated from the full equations (1) and (2) and from the volume flow routing equation (7).

A method for feed-forward control and gate stroking

In previous work on gate stroking, programs have been written which solve the long wave equations backwards in space and time, given the complete desired flow and surface history at the downstream end. There have been many difficulties encountered, and we believe that these are explained in view of the diffusive nature of the system. The theory presented above shows that in a typical canal, the downstream wave is substantially diffused, and this is evidence for our explanation of the difficulties encountered by gate stroking calculations. As a finite change at the upstream end creates a slower and more diffuse effect downstream, if one proceeds in the reverse direction, any irregularities or rapid transients downstream quickly grow into huge fluctuations when the upstream conditions necessary to bring them about are computed. Cunge *et al.* (1980) have criticised the very concept of computing gate stroking in similar terms.

Here we attempt a different approach to gate stroking, whereby we use existing programs that solve the equations in the usual manner, forward in time and down the canal. We use the full dynamic long wave equations, although we could have used the low-Froude approximation of the previous section. A method based on linear systems theory then gives us a method of performing fast flow routing computations without having to invoke the usual full solution techniques, and gives a method that can solve the feed-forward control problem. The method is technically a form of this form of control rather than gate stroking, as we do not impose conditions on the water surface at the downstream gate. Were we so to do, it would properly be termed gate stroking. However, the differences are not very important. The results demonstrate graphically how either operation has fundamental difficulties.

Our approach is mathematically justified only for systems that are linear. The governing long wave equations that we use, where products and nonlinear functions of the flow variables appear, are not linear. In many irrigation applications, however, changes about a base state of flow are usually quite small, and the system behaves very nearly linearly.

Fundamental ideas from linear systems theory have been extensively used in hydrology, for example in the form of the unit hydrograph (Chow *et al.*, 1988). Dooge (1973) and Keefer (1976) amongst others have used this approach in the study of waves in canals. The output from a system, in this case the outflow at the downstream regulator, can be written in terms of the inflow at the other end, expressing it at a point in time as a weighted integral of the input. The weight function in that integral is the transfer or system function which expresses the effect downstream of a single unit of flow increase at the upstream end. Here we write it in discrete form, replacing the integrals by sums. We suppose that the transfer function of the system is the sequence of K numbers h_k for $k = 0,1,\ldots,K-1$, such that if the input to the system were a single spike of flow at time 0, then the values of the h_k would be the resultant outflow hydrograph.

One could use the advection-diffusion approximation to obtain a theoretical transfer function for the system, however here we prefer to obtain it directly from a solution of the full long wave equations. We used the program described above which solves the full equations accurately, and applied it to the model single pool described above. We considered a base flow of $10\ \mathrm{m}^3/\mathrm{s}$. The inflow was increased smoothly (a Gaussian function of time) by 25% up to a maximum of $12.5\ \mathrm{m}^3/\mathrm{s}$ and back down to the base flow over a period of about three hours. The program then simulated conditions in the canal for a total of 20 hours.

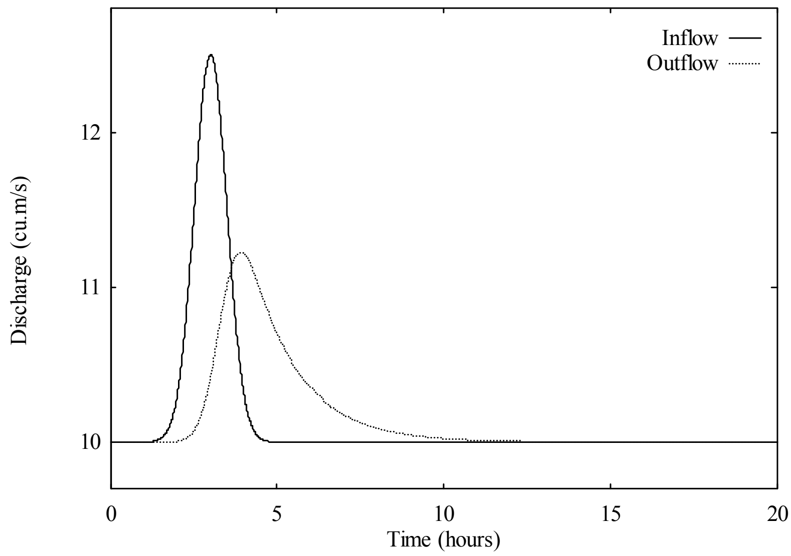

Figure 3: Specified inflow hydrograph and computed outflow hydrograph

Results are shown in Figure 3. Even in this relatively short length of waterway the "wave" has diminished by a factor of about half, and the time over which the change in discharge is felt at the outlet is considerably greater. In fact, it is about eight or nine hours before the system has returned to steady state. Naively, for a canal 7km long, one might feel that the wave that arrived at the outlet would be substantially the same as that which was generated at the inlet.

We computed the transfer function h_k using standard procedures that are described in Fenton, Aughton, and Oakes (1998). The results are shown in Figure 4. The sequence of points shown (the lines between them have no significance) corresponds to the outflow due to a single inflow spike of magnitude unity at time zero. The maximum value of the transfer function is about 0.35. The kind of behaviour we see in the figure seems to correspond with that observed in Figure 3, where the hump diffuses considerably and its influence is felt for a long time at the outlet. The sequence composed of the h_k could be used to provide a rapid means of simulation if they were convolved with a sequence of points corresponding to an actual input sequence.

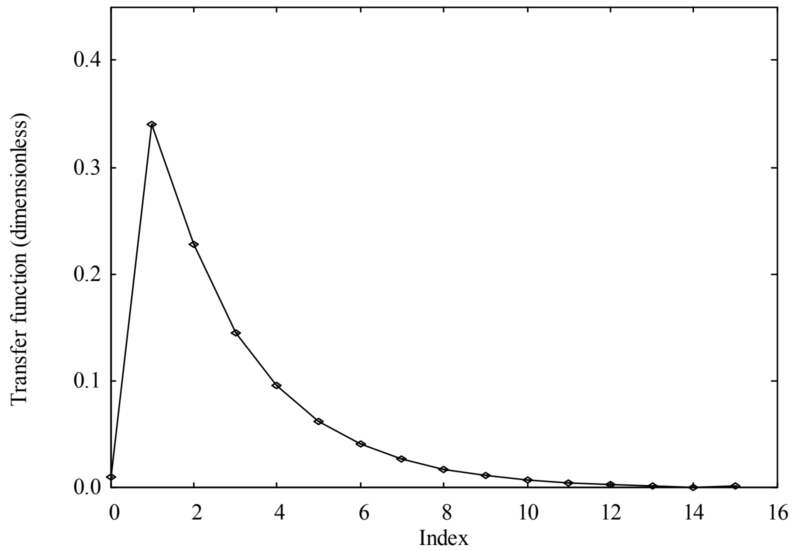

Figure 4: Transfer function h_k for computing outflow from inflow

With a view to developing a method for feed-forward control and gate stroking we then computed the inverse transfer function u_k, by simply reversing the roles of input and output. The results are as shown in Figure 5, where the transfer function oscillates wildly with magnitudes between –8 and +6, very much greater than for the forward transfer function. Some consideration of the mechanics, however, shows that this result might well be expected. The forward transfer function in Figure 4 shows the outflow caused by a single inflow spike of magnitude unity, with the expected decay and time shift. The inverse transfer function in Figure 5, however, shows the inflow that would cause a single *outflow* spike of magnitude unity for subsequent use in gate stroking calculations. Given the diffusive nature of the system as seen in the figures, it can be imagined that it would require a fairly remarkable input which would travel and diffuse such that all the variation combined to produce a single hump in the downstream hydrograph.

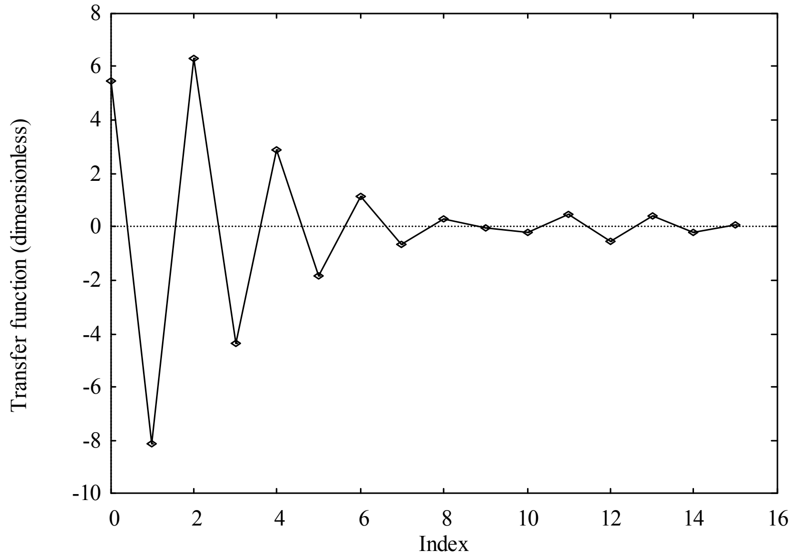

Figure 5: Inverse transfer function u_k for computing the gate stroking solution

This does not mean that the solution is invalid, however. We tested it for some typical required outputs, consisting of a single smooth increase in flow from the base level, given by a tanh function. Firstly, we used the function

$$\frac{Q}{Q_0} = \frac{1}{2}\left(1 + \tanh\left(1.5\left(T - 6\right)\right)\right), \qquad (10)$$

where T is the time in hours. This function increases from 0 to 1 continuously, with 90% of the change of flow occurring in a period of 1-2 hours, centred at 6 hours. The results are shown in Figure 6, where the dashed line shows the desired outflow (equation (10)), and the solid line shows the computed feed-forward control solution. Huge variations of flow upstream are required by the change downstream. They tend to cancel each other as the wave propagates downstream so that the process of frictional diffusion ultimately leads to the required single smooth increase over a finite time. These are the sort of oscillations reported by many other workers in the area of gate stroking, as noted above.

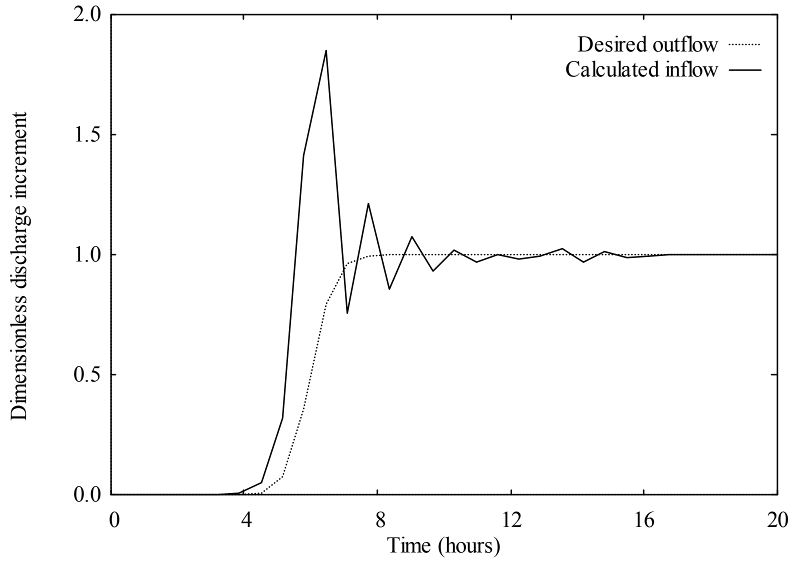

Figure 6: Outflow desired and the corresponding inflow required by the gate stroking solution where 90% of the flow increase occurs within 1.5 hours

Figure 7 shows the same magnitude of flow increase but where it takes place over a period of about 4.5 hours, three times that of Figure 6 (the factor of 1.5 in the argument of the tanh function in equation (10) was reduced to 0.5). The situation for the upstream gate is now very different, for it can bring about the required downstream hydrograph with very little unnecessary motion.

While it might be thought that this is a relatively satisfactory state of affairs, further computations soon dispel that illusion. Considering a canal twice the length gave an inverse transfer function that oscillated even more wildly than that shown in Figure 5, varying between values of ± 40! Of course, if one required a change downstream taking place in a matter of minutes, huge fluctuations would ideally be required.

These results suggest that the feed-forward / gate stroking problem is indeed flawed, and that it is unreasonable to require the precise satisfaction of flows at the downstream boundary by upstream gate movements. However, there is no reason not to use upstream movements to bring about an approximate solution, such as that shown in Figure 7 compared with that of Figure 6.

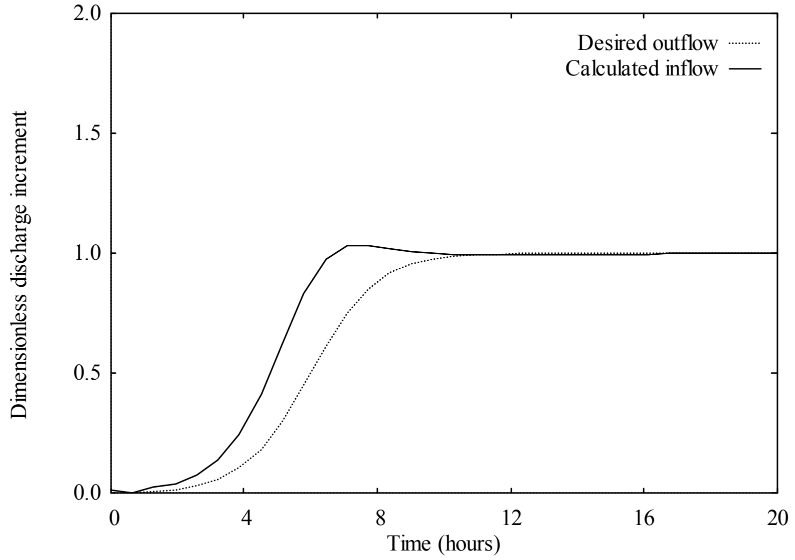

Figure 7: As for Figure 6 but where 90% of the increase occurs within 4.5 hours

Conclusions

We have considered the equations governing the motion of waves in canals and have shown that a low-Froude number approximation gives an equation which can be solved numerically to simulate wave motion in canals rather more simply than existing methods. More work needs to be done to develop faster solution methods, but its simplicity may make it a useful method for canal systems. The equation shows explicitly how diffusion due to friction is an important feature of wave motion in irrigation canals, although they are often relatively smooth and have a mild slope. We have shown that it is this feature which makes feed-forward control and gate-stroking very difficult. As disturbances downstream are damped considerably and their effects smeared out in time, to specify a downstream variation which might vary rapidly in time must require huge fluctuations at the upstream end. This explains the results of previous writers who have attempted to implement gate-stroking programs and who invariably found large fluctuations necessary.

Gate stroking measures can be undertaken for canals that are not too long, but in general, the precise satisfaction of the downstream requirements by upstream gate operations alone seems to be not possible. Rather, upstream gate operations should be used to satisfy approximately the downstream requirements and control measures used on the downstream gate so as to achieve the flows required while attempting to minimise surface disturbances.

References

Chow, V. T. and Maidment, D. R. and Mays, L. W. (1988) *Applied Hydrology*, McGraw-Hill, New York.

Clemmens, A. J., C. M. Burt and D. C. Rogers (1995) Introduction to canal control algorithm needs, *Proc. First Int. Conf. Water Resources Engnng, San Antonio, Texas,* Vol 1, Eds. W. H. Espey and P. G. Combs, ASCE, New York, 1-5.

Clemmens, A. J., T. F. Kacerek, B. Grawitz and W. Schuurmans (1998) Test cases for canal control algorithms, *J. Irrign and Drainage Engng* **124**, 23-30.

Cunge, J. A., Holly, F. M., and Verwey, A. (1980), *Practical Aspects of Computational River Hydraulics*. Pitman, London.

Dooge, J. C. I. (1973) *Linear theory of hydrologic systems*, Technical Bulletin, 1468, United States Department of Agriculture, Washington, D.C..

Fenton, J. D. (1999) On the long wave equations, manuscript submitted for publication.

Fenton, J. D., Aughton, D. and Oakes, A. (1998), On gate stroking and upstream and downstream control, *Proceedings, Irrigation Association of Australia 1998 National Conference, Brisbane*, 609-618.

Keefer, T. N. (1976), Comparison of linear systems and finite difference flow-routing techniques, *Water Resources Res.* **12(**5), 997-1006.

Rubicon (1998) Draft Report on Study of SCADA Operation of Pyramid Hill No 1 Channel, Rubicon Systems Australia, Hawthorn East.

Singh, V. P. (1996) *Kinematic wave modeling in water resources: surface-water hydrology*, Wiley.

Strelkoff, T. S. and A. J. Clemmens (1998) Nondimensional expression of unsteady canal flow, *J. Irrigation and Drainage Engng* **124**, 59-62.

Wylie, E. B. (1969), Control of transient free-surface flow, *J. Hyd. Div., ASCE* **95**(1), 347-361.