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A SPECTRAL METHOD FOR DIFFRACTION PROBLEMS

John D. Fenton

Department of Mechanical Engineering, Monash University,
Clayton, Victoria, Australia 3168

ABSTRACT

The conventional method of solving the problem of the scattering of waves by a large structure of arbitrary shape and calculating the forces induced is to solve an integral equation over the surface of the body. This paper describes a more direct approach, in which a series solution for the potential due to incident and scattered waves is written, and coefficients in the series found by satisfying simultaneously the boundary condition at a number of points on the body. Results are obtained for structures which are axisymmetric about a vertical axis. These suggest that the method has some limitations and some exciting possibilities which make it worth developing for more general problems.

INTRODUCTION

Offshore structures with dimensions which are a significant fraction of the wavelength scatter the incident waves, and a significant component of the force on the structure is due to that diffraction of the waves. For structures which have relatively simple geometry such as truncated circular cylinders it is possible to develop analytical methods, or methods which make extensive use of series of known solutions, each term of which satisfies the field equation. Variational techniques or Galerkin techniques may be used (see, for example, Black, 1975), however these methods have been limited to simple geometries. For bodies of arbitrary geometry, methods based on boundary integral equations have been the usual method of solution (Sarpkaya and Isaacson, 1981), using a time dependent Green's function, which contains an infinite series of Bessel functions, and then integrating this Green's function over the surface with an unknown potential or source strength to give the integral equation. For bodies which are axisymmetric, the problem can be decomposed into Fourier modes and each solved separately. The author (Fenton, 1978) showed how the convergence of the series could be enhanced if the singularities in the Green's function were removed. The result was a very complicated function indeed, and Isaacson (1982) showed that there were some mathematical and typographical errors in that paper.

This paper is an initial attempt to develop a method for general bodies which is simpler than integral equation methods, because, like the variational and Galerkin methods, it assumes a solution which satisfies the field equation and the surface boundary condition, and the problem remains simply to find the coefficients in the series by satisfying the boundary condition on the body at a sufficient number of points. The only numerical approximation of the method is in the truncation of the series of solutions. Initially only bodies which occupy the entire depth of the water can be treated, however in further work a more general method will be developed. The method is further developed for axisymmetric structures and some problems solved. It is simple to implement, although the equations obtained are not as robust as those based on integral equations. However, the numerical properties of the system of equations can be considerably improved by performing a discrete numerical transform. For practical problems, the nature of the approximation of the present method may have some advantages, as it somehow mimics the nature of the diffraction process, that higher order terms are relatively unimportant, such that the wave may not "see" the fine details of a structure.

Results which are presented show that the method in its present form is capable of high accuracy, but for difficult geometries it may be better to use a different set of basis functions. In its present stage of development, the method has some limitations and some exciting possibilities, including extension to nonlinear problems.

FORMULATION

Consider the incident wave field

$$\Phi = \Re(\phi e^{-i\sigma t})$$

where Φ is the velocity potential such that the velocity $\mathbf{u} = \nabla\Phi$, $\Re()$ shows that the real part is to be taken, $\phi = \phi_i + \phi_s$, the complex potential which is the sum of the incident and scattered wave fields respectively, $\sigma = 2\pi/T$ is the angular frequency and T is the wave period, and t is time. We assume that the incident potential due to waves of wavenumber $k = 2\pi/L$, where L is the wavelength, and height H is

$$\phi_i = \frac{-igH}{2\sigma}\frac{\cosh k(z+d)}{\cosh kd}e^{ikx},$$

where $i = \sqrt{-1}$, g is gravitational acceleration, and x is a cartesian coordinate axis in the direction of the waves propagation, z is vertically upwards with origin at the mean water level, and d is the water depth. The expression can be written

$$\phi_i = \frac{-igH}{2\sigma}\frac{\cosh k(z+d)}{\cosh kd}\sum_{m=0}^{\infty}\beta_m J_m(kr)\cos m\theta,$$

where $J_m()$ is a Bessel function of the first kind, (Abramowitz and Stegun, 1965), $\beta_0 = 1$, and $\beta_m = 2i^m$ for $m \ge 1$. The polar coordinates (r,θ) have origin at the cylinder axis and the x axis respectively. This formulation follows that given, for example, in Sarpkaya and Isaacson (1981) or Isaacson (1982).

In this work we will be considering structures which occupy the whole of the vertical axis below the surface. Hence we assume a form for the scattered wave of

$$\phi_s = \frac{-igH}{2\sigma} \sum_{m=0}^{\infty} \cos m\theta \left(a_{m0} \frac{\cosh k(z+d)}{\cosh kd} H_m(kr) + \sum_{j=1}^{\infty} a_{mj} \cos \kappa_j(z+d) K_m(\kappa_j r) \right),$$

where the a_{mj} for $j, m = 0, \ldots, \infty$ are coefficients to be determined, the $K_m()$ are modified Bessel functions, which go to infinity as $r \to 0$, hence they are ruled out locally if the body does not occupy all of the sub-surface vertical axis. The $H_m()$ are Hankel functions, also singular at the origin, and the κ_j are solutions of the linear dispersion relation:

$$\sigma^2 = gk \tanh kd = -g\kappa_j \tan \kappa_j d.$$

In practice it is necessary to solve this equation numerically for sufficient κ_j, $j = 1, \ldots$, for which McKee (1988) has presented a simple and accurate method.

The problem is now to satisfy the boundary condition at points on the body, that the velocity normal to the structure u_n is zero, that is

$$u_n = u_r n_r + u_\theta n_\theta + u_z n_z = 0,$$

where we have used cylindrical coordinates in which n_r, n_θ, n_z are the components of the unit normal vector at a point and the three components of velocity are

$$u_r = \frac{\partial \phi}{\partial r}, \quad u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}, \quad u_z = \frac{\partial \phi}{\partial z}.$$

Hence, the velocity components of the incident wave $(u_{i,r}, u_{i,z}, u_{i,\theta})$ are

$$u_{i,r} = \frac{-igH}{2\sigma} k \frac{\cosh k(z+d)}{\cosh kd} \sum_{m=0}^{\infty} \beta_m J'_m(kr) \cos m\theta,$$

$$u_{i,\theta} = \frac{igH}{2\sigma} \frac{1}{r} \frac{\cosh k(z+d)}{\cosh kd} \sum_{m=0}^{\infty} m\beta_m J_m(kr) \sin m\theta,$$

$$u_{i,z} = \frac{-igH}{2\sigma} k \frac{\sinh k(z+d)}{\cosh kd} \sum_{m=0}^{\infty} \beta_m J_m(kr) \cos m\theta.$$

The components of velocity of the scattered wave $(u_{s,r}, u_{s,z}, u_{s,\theta})$ are:

$$u_{s,r} = \frac{-igH}{2\sigma} \sum_{m=0}^{\infty} \cos m\theta \left(a_{m0}\, k \frac{\cosh k(z+d)}{\cosh kd} H'_m(kr) + \sum_{j=1}^{\infty} a_{mj} \kappa_j \cos \kappa_j(z+d) K'_m(\kappa_j r) \right),$$

$$u_{s,\theta} = \frac{igH}{2\sigma} \frac{1}{r} \sum_{m=0}^{\infty} m \sin m\theta \left(a_{m0} \frac{\cosh k(z+d)}{\cosh kd} H_m(kr) + \sum_{j=1}^{\infty} a_{mj} \cos \kappa_j(z+d) K_m(\kappa_j r) \right),$$

$$u_{s,z} = \frac{-igH}{2\sigma} \sum_{m=0}^{\infty} \cos m\theta \left(a_{m0} k \frac{\sinh k(z+d)}{\cosh kd} H_m(kr) - \sum_{j=1}^{\infty} a_{mj}\kappa_j \sin \kappa_j(z+d) K_m(\kappa_j r) \right).$$

Satisfying the boundary condition on the cylinder gives the equation, after dropping common factors and multiplying by d to make it non-dimensional:

$$\begin{aligned} &n_r \left(\sum_{m=0}^{\infty} \cos m\theta \left(kd \frac{\cosh k(z+d)}{\cosh kd} (\beta_m J'_m(kr) + a_{m0} H'_m(kr)) + \sum_{j=1}^{\infty} a_{mj} \kappa_j d \cos \kappa_j(z+d) K'_m(\kappa_j r) \right) \right) \\ &- \frac{n_\theta d}{r} \left(\sum_{m=0}^{\infty} m \sin m\theta \left(\frac{\cosh k(z+d)}{\cosh kd} (\beta_m J_m(kr) + a_{m0} H_m(kr)) + \sum_{j=1}^{\infty} a_{mj} \cos \kappa_j(z+d) K_m(\kappa_j r) \right) \right) \\ &+ n_z \left(\sum_{m=0}^{\infty} \cos m\theta \left(kd \frac{\sinh k(z+d)}{\cosh kd} (\beta_m J_m(kr) + a_{m0} H_m(kr)) - \sum_{j=1}^{\infty} a_{mj} \kappa_j d \sin \kappa_j(z+d) K_m(\kappa_j r) \right) \right) = 0, \end{aligned}$$

in which the co-ordinates (r,θ,z) are not independent but are such that they correspond to a point on the surface of the body. This equation for that point on the body involves all the unknown a_{mj}. To solve for those coefficients we have to consider a sufficient number of such points, giving a linear system of equations.

EQUATIONS FOR AXISYMMETRIC BODIES

In this work we further limit ourselves to bodies which are axisymmetric about the vertical axis, such that $n_\theta = 0$, and all the geometric quantities r, z, n_r, and n_z are independent of θ such that we can satisfy each term in m in the Fourier series separately, and so taking and combining the coefficients of $\cos m\theta$ we have the equation:

$$n_r\left(kd\frac{\cosh k(z+d)}{\cosh kd}(\beta_m J'_m(kr) + a_{m0} H'_m(kr)) + \sum_{j=1}^{\infty} a_{mj}\,\kappa_j d\,\cos\kappa_j(z+d)K'_m(\kappa_j r) \right)$$

$$+n_z\left(kd\frac{\sinh k(z+d)}{\cosh kd}(\beta_m J_m(kr) + a_{m0} H_m(kr)) - \sum_{j=1}^{\infty} a_{mj}\kappa_j d\sin\kappa_j(z+d)\,K_m(\kappa_j r) \right) = 0,$$

for each $m = 0, 1, \ldots$. Rearranging, the equation to be satisfied for each m is

$$a_{m0}\,kd\left(n_r\frac{\cosh k(z+d)}{\cosh kd}H'_m(kr) + n_z\frac{\sinh k(z+d)}{\cosh kd}H_m(kr) \right)$$

$$+\sum_{j=1}^{N-1} a_{mj}\,\kappa_j d\,(n_r\ \cos\kappa_j(z+d)\,K'_m(\kappa_j r) - n_z\ \sin\kappa_j(z+d)\,K_m(\kappa_j r))$$

$$= -\beta_m kd\left(n_r\frac{\cosh k(z+d)}{\cosh kd}J'_m(kr) + n_z\frac{\sinh k(z+d)}{\cosh kd}J_m(kr) \right), \qquad \text{(A)}$$

where, for computational purposes, the summation over j has been truncated at N-1 rather than infinity. The right side of the equation can be evaluated for any point on the body. The left side, for each value of m, contains a total of N unknowns, the a_{mj} for $j = 0, \ldots, N-1$, for which the equations must be solved simultaneously.

Writing equation (A) at M separate points, (r_i, z_i), $i = 0, \ldots, M-1$, where the integer symbol i should not be confused with the same symbol for $\sqrt{-1}$, the system can be written as the complex matrix equation

$$[A_{ij}]\ \mathbf{a} = \mathbf{b}, \qquad \text{(B)}$$

where the elements of the $M \times N$ matrix A_{ij} are given by:

$$A_{i0} = kd\left(n_r(i)\frac{\cosh k(z_i+d)}{\cosh kd}H'_m(kr_i) + n_z(i)\frac{\sinh k(z_i+d)}{\cosh kd}H_m(kr_i) \right), \text{ and}$$

$$A_{ij} = \kappa_j d(n_r(i)\ \cos\kappa_j(z_i+d)K'_m(\kappa_j r_i) - n_z(i)\ \sin\kappa_j(z_i+d)K_m(\kappa_j r_i)), \text{ for } j = 1, \ldots N-1,$$

for $i = 0, \ldots M-1$. The N-vector of unknowns is $\mathbf{a} = [a_{m0}\ a_{m1}\ \ldots\ a_{m,N-1}]^{\mathrm{T}}$, and the M-vector of right hand sides is $\mathbf{b} = [b_i]$, where

$$b_i = -\beta_m kd\left(n_r(i)\frac{\cosh k(z_i+d)}{\cosh kd}J'_m(kr_i) + n_z\frac{\sinh k(z_i+d)}{\cosh kd}J_m(kr_i) \right).$$

COMPUTATIONAL CONSIDERATIONS

The matrix is not dominated by diagonal terms, unlike those equations obtained from boundary integral equations, where numerical solution seems to present no real problems. In fact, the $K_m()$ show exponential decay for large values of the argument, and it is possible that the magnitudes of the coefficients in the present theory vary by orders of magnitude, which can render the matrix ill-conditioned.

Fortunately modern methods of solution can handle systems of equations which are poorly conditioned. A good example is the Singular Value Decomposition method (Press *et al.*, 1992) which almost always provides a useful numerical answer, and is particularly helpful in providing the least-squares solution to an over-determined set of linear equations. In the first development of this work an equation was written for a significantly larger number of boundary points than the number of coefficients available, and that system was solved in a best (least squares) sense by the Singular Value Decomposition program provided by Press *et al.*. This was found to not as computationally robust as would be necessary in practice.

An alternative approach is suggested by previous work in water wave theory, which is the use of a numerical integral transform. It is known that the set of functions $\{\cosh k(z+d), \cos\kappa_j(z+d) \text{ for } j=0,\ldots,N-1\}$ is orthogonal when integrated in the vertical, using the boundary condition on the bottom and the linear dispersion relation on the surface, such that if any two *different* functions are integrated, the result is zero. This suggests that in the present work, a convenient trick might be to use a numerical discrete transform of the system of equations, even if the computational points (r_i, z_i) are not in a vertical line, in some rough way to mimic the effects of a full integral transform in a vertical line. This may give a more robust system of equations.

This transform can be represented as premultiplying each side of the matrix equation (B) by the $N\times M$ matrix T, where

$$T=\begin{bmatrix} \frac{1}{2}\frac{\cosh k(z_0+d)}{\cosh kd} & \frac{\cosh k(z_1+d)}{\cosh kd} & \cdots \\ \frac{1}{2}\cos\kappa_1(z_0+d) & \cos\kappa_1(z_1+d) & \cdots \\ \frac{1}{2}\cos\kappa_2(z_0+d) & \cdots & \cdots \\ \cdots & \cdots & \frac{1}{2}\cos\kappa_{N-1}(z_{M-1}+d) \end{bmatrix}.$$

The factor of 1/2 in the first column, associated with point z_0, is to mimic the act of integration by representing the sum as a trapezoidal sum. The same factor multiplies the last column. Instead of the over-determined $M\times N$ system to be solved in a least squares sense, the result is a smaller $N\times N$ system in which it is required to satisfy all the boundary condition at all the points, but combined linearly.

In practice it was found that this procedure did not work particularly effectively. It was found, however, that if the Bessel functions were also included in the numerical transform, mimicking a Galerkin approach, the procedure was very much improved, and the convergence of the solution was much more rapid. The premultiplying matrix used was

$T =$

$$\begin{bmatrix} \frac{1}{2}\frac{\cosh k(z_0+d)}{\cosh kd} J_m(kr_0) & \frac{\cosh k(z_1+d)}{\cosh kd} J_m(kr_1) & \cdots \\ \frac{1}{2}\cos\kappa_1(z_0+d)\,K_m(\kappa_1 r_0) & \cos\kappa_1(z_1+d)\,K_m(\kappa_1 r_1) & \cdots \\ \frac{1}{2}\cos\kappa_2(z_0+d)\,K_m(\kappa_2 r_0) & \cdots & \cdots \\ \cdots & \cdots & \frac{1}{2}\cos\kappa_{N-1}(z_{M-1}+d)\,K_m(\kappa_{N-1} r_{M-1}) \end{bmatrix}.$$

All results presented in this paper are for the case where the system of equations has been numerically transformed by premultiplication of both sides and the SVD algorithm used.

CALCULATION OF BODY FORCES

From irrotational theory, the expression for the pressure p at any point in the fluid is

$$p = -\rho\left(gz + \frac{\partial \Phi}{\partial t}\right),$$

(Sarpkaya and Isaacson, 1981). The force on the body $\mathbf{F}$ is given by

$$\mathbf{F} = -\int_S p\,\hat{\mathbf{n}}\,dS,$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the local body surface and directed into the fluid, and dS is an element of surface area of the body. The moment on the body $\mathbf{M}$ is given by

$$\mathbf{M} = -\int_S p\,\mathbf{r}\times\hat{\mathbf{n}}\,dS,$$

where $\mathbf{r}$ is the vector from the point about which the moment is desired. Here we consider only the wave-induced loading by ignoring the hydrostatic component of the pressure. We introduce a set of unit vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ in the (x, y, z) coordinate directions respectively, such that

$$\hat{\mathbf{n}} = \mathbf{i}\,n_r\,\cos\theta + \mathbf{j}\,n_r\,\sin\theta + \mathbf{k}\,n_z,$$

and use $dS = r\,d\theta\,ds$, where ds is an element of arc length of the body of revolution in the (r, z) plane. Substituting the equations necessary and performing the integration with respect to θ, all other contributions from the Fourier series other than the zeroth and the first disappear, giving the result

$$\mathbf{F}(t) = \Re\left(-\pi\rho g H e^{-i\sigma t}\int_C \left(\mathbf{i}\,\tfrac{1}{2}\phi_1(r(s), z(s))\,n_r(s) + \mathbf{k}\,\phi_0(r(s), z(s))\,n_z(s)\right) r(s)\,ds\right),$$

where the domain of integration is the wetted arc of the body in the (r, z) plane, and where the coefficients ϕ_0 and ϕ_1 are dimensionless coefficients in the series for ϕ:

$$\phi = \frac{-igH}{2\sigma}\sum_{m=0}^{\infty}\phi_m(r, z)\,\cos m\theta\ ,$$

such that

$$\phi_m(r, z) = \frac{\cosh k(z+d)}{\cosh kd}\left(\beta_m J_m(kr) + a_{m0}\,H_m(kr)\right) + \sum_{j=1}^{N-1} a_{mj}\,\cos\kappa_j(z+d)\,K_m(\kappa_j r).$$

This shows the fortunate result that to obtain the force on a body we need only solve two problems, for $m=0$ and $m=1$, each involving $N+1$ unknowns. This is enough also to obtain the moment about the base, for similar operations to that for the force give

$$\mathbf{M}(t) = \Re\left(-\pi\rho g H e^{-i\sigma t}\int_C \mathbf{j}\,\tfrac{1}{2}\phi_1(r(s),z(s))\,(r(s)n_z(s) - z(s)\,n_r(s))r(s)\,ds\right).$$

RESULTS

Vertical circular cylinder

In the case where the body is a vertical cylinder of radius a, the solution of (A) is

$$a_{m0} = -\beta_m \frac{J_m'(ka)}{H_m'(ka)} \quad \text{and} \quad a_{mj} = 0 \quad \text{for all } j > 0,$$

for all m, giving the well-known solution of MacCamy and Fuchs (1954). This was programmed on a computer and the matrix problem solved numerically for various values of N. In each case the double precision computer program obtained all the coefficients and the forces correct to more than six figures of accuracy.

Frustrum of cone

A simple geometry is that described by Sarpkaya and Isaacson (1981, p464) and Isaacson (1982, p192), which is a conical structure extending through the water surface. Results for the forces are shown in Figure 1. If the results were to be compared with graphs shown in the two references cited here, it can be seen that the results agree closely.

It was found for this case of a bulk body with no appurtenances or fine structural detail that very low levels of spectral approximation (few terms in the series) can give a good engineering approximation to the force for both the raw overdetermined set of equations and the transformed equations. However, as the number of terms in the series was increased beyond about half the number of boundary condition points the behaviour of the spectral method without the transform of the system of equations was quite unreliable, and apparent convergence could be reached to values considerably in error. The transformed system of equations was very robust indeed, and its convergence was very rapid and satisfactory. The author has been surprised how few terms in the series it is necessary to describe the scattering problem for such bodies to acceptable accuracy.

Compound cylinder

A different picture emerges if the compound cylinders shown in Figure 2 are studied. The words "square" and "quadrant" refer to the shape in section. Rotating about the axis of symmetry gives a cylinder and hemisphere respectively. It was found that the present method did not perform as well in some of these cases. Here the flow around the bodies is complicated, partly by the tower on top of the base having a different length scale from that of the base, which requires more spectral terms to handle it, and partly by the sharp corner in the flow, as shown in Figures 2(a) and (b), which according to irrotational flow theory has a singularity there. The author presumes that boundary integral methods make no special allowance for such singularities.

Results for the horizontal force are presented in Figure 3. The method was quite satisfactory for (a), where the ratio of tower radius to base radius was 1/2, but for (b), with a value of 1/4, the convergence of the spectral method was not as good, as can be seen in Figure 3, requiring more terms to describe the scattering due to the base and that due to the tower, while making use of the $K_m()$ Bessel functions which go to infinity at the origin. If the corner of the base was rounded as in Figure 2(c), the flow singularity no longer existed, and the method worked much better. A more extreme case is that of the smooth hemispherical base in (d), where very few terms were needed to describe the flow. It is interesting that avoiding a bluff front on the body seems to reduce the force on it considerably, which seems obvious but which the author has not seen discussed.

Finally, Figure 4 shows the flow field around two of the structures, and how the spectral method makes detailed flow computations relatively simple. Examination of (a) shows that the method is not very good at satisfying the boundary condition near the sharp corner, but then neither would the non-separated irrotational assumption be valid in this region. Neither, one must say, would the wave field be too bothered about the details of such a fine structure below the surface. In a sense, the present method approximates the problem in a similar way to that of the waves, that fine subsurface structural details play no great role in determining the loading. In the case of the hemisphere in Figure 2(d), Figure 4(b) shows that the streamlined shape allows fluid to pass over the structure at some speed, so that stagnation pressure effects, and force, are smaller. The lack of a singularity meant that the present spectral method performed very well, with few terms in the series.

REFERENCES

Abramowitz, M. and I.A. Stegun 1965. *Handbook of Mathematical Functions*, Dover, New York.

Black, J.L. 1975. Wave forces on large bodies, A survey of numerical methods, *Proc. Civil Engineering in the Oceans III,* ASCE, *pp*. 924-938.

Fenton, J.D. 1978. Wave forces on vertical bodies of revolution, *J. Fluid Mech.* 85, 241-255.

Isaacson, M. de St. Q. 1982. Fixed and floating axisymmetric structures in waves, *J.Waterway Port Coastal and Ocean Div. ASCE* 108, 180-199.

MacCamy, R.C. and R.A. Fuchs 1954. Wave forces on piles: a diffraction theory. U.S. Army Corps of Engineers, Beach Erosion Board, Technical Memorandum No. 69.

McKee, W.D. 1988. Calculation of evanescent wave modes, *J. Waterway Port Coastal and Ocean Engnrng.* 114, 373-378.

Press, W.H., S.A.Teukolsky, W.T. Vetterling and B.P. Flannery, 1992. *Numerical Recipes in C, Second Edition*, C.U.P., Cambridge. An edition with Fortran programs also exists.

Sarpkaya, T. & M. de St.Q. Isaacson 1981. *Mechanics of Wave Forces on Offshore Structures*, Van Nostrand Reinhold, New York.

FIGURES

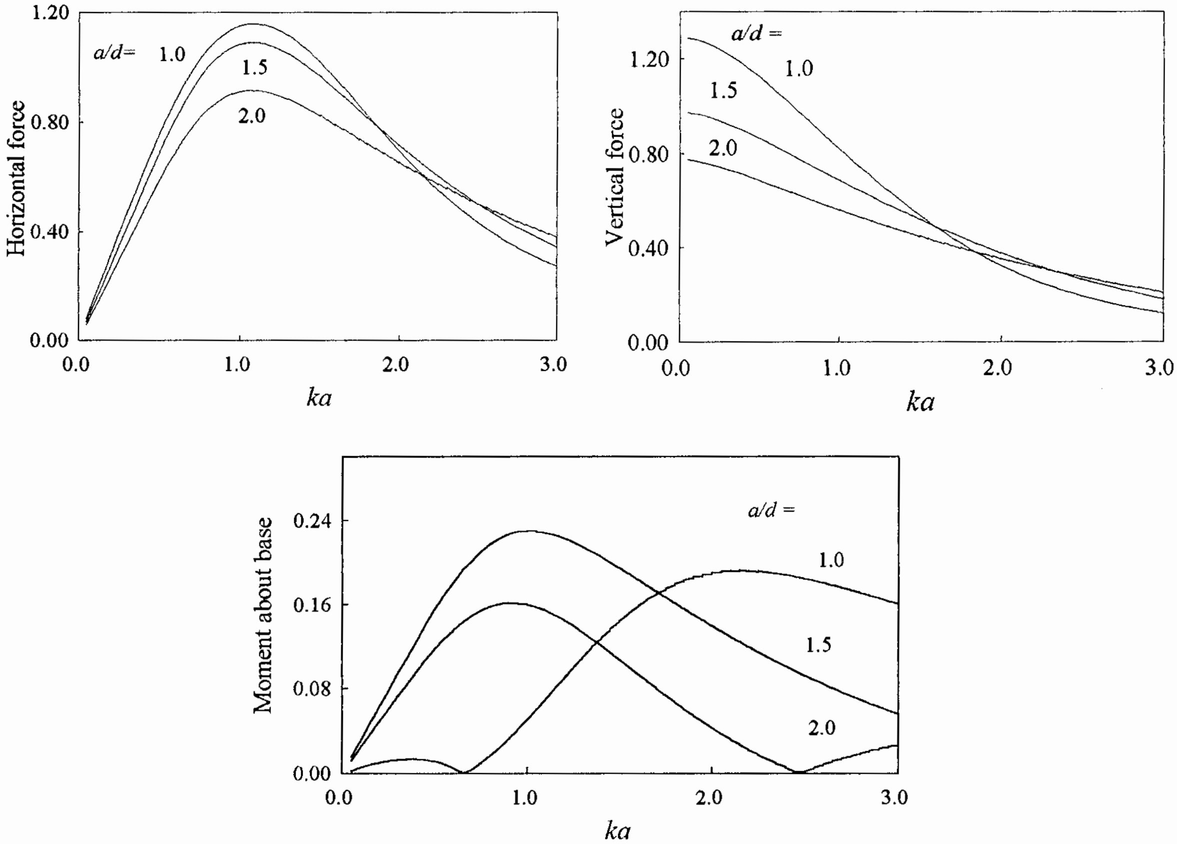


Figure 1: Forces on surface-piercing cone with base radius $a/d = 1.0$, 1.5 and 2.0, for a cone slope of 60°. Horizontal force plotted is $F_x/\rho gHa^2$, vertical force is $F_z/\rho gHa^2$, and moment about base is $M_y/\rho gHa^3$.

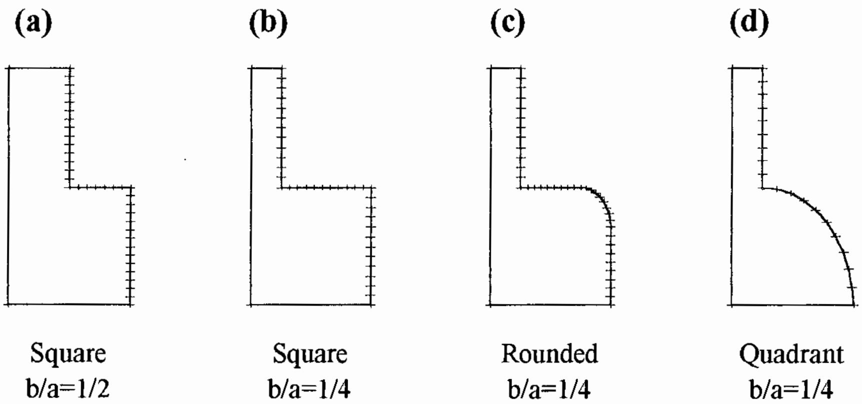


Figure 2: Compound tower structures studied numerically. Height and radius of the central base was half the water depth.

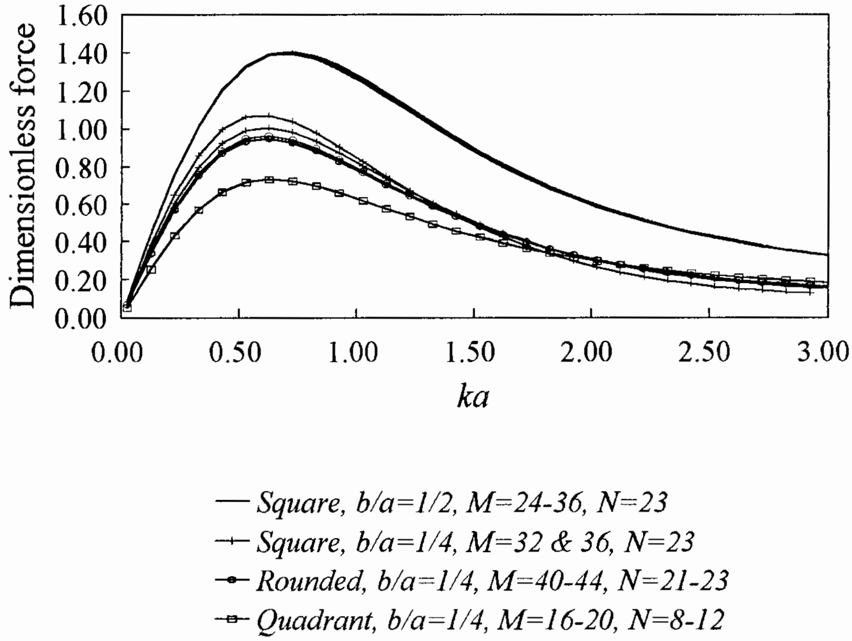
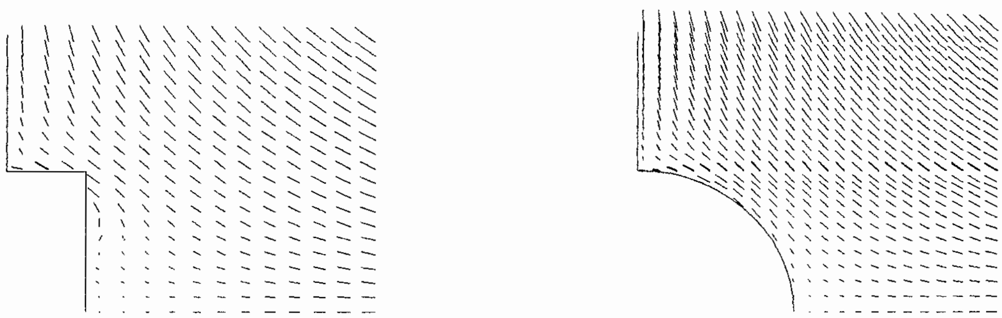


Figure 3: Results for horizontal force $F_x/\rho gHa^2$ on compound tower structures of base radius a and tower radius b



(a) **(b)**

Figure 4: Velocity fields due to the $m = 0$ component in the vertical plane ahead of two of the compound towers. (a) the pedestal square in section, $b/a = 3/4$, (b) the quadrant section, $b/a = 1/4$.