Reservoir routing

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Abstract This paper asserts that the traditional method for reservoir routing is unnecessarily complicated. It requires the solution of a transcendental equation at each time step. Reservoir routing is actually simply the numerical solution of a differential equation. Any standard method can be used, and all are simpler than the traditional method. The paper also shows that the alternative form of the governing equation in terms of the reservoir surface elevation has some advantages over the usual form involving storage volume. The presentation incorporates the case where reservoir outflow may be varied by control of valves or spillway gates. Numerical methods for reservoir routing are examined and compared, and it is concluded that simple standard methods for solving differential equations are to be preferred to the traditional method for flood routing, and should replace it.

Laminage des débits de crues par un réservoir

Résumé Cette communication part du principe que la méthode traditionnelle, utilisée pour calculer le laminage produit par un réservoir, a masqué la simplicité fondamentale du problème à résoudre. En revanche, s'il est reconnu que le calcul du laminage correspond au processus de la résolution d'une équation différentielle, alors il est possible d'employer les méthodes numériques normales, celles-ci étant beaucoup plus simples que la méthode traditionnelle. En outre, il est démontré qu'une autre forme d'équation principale, exprimée en fonction du niveau de l'eau d'un réservoir, présente certains avantages par rapport à l'équation habituellement employée pour calculer le volume d'eau accumulée. Un partie du présent exposé est consacrée au problème du réservoir dont le débit peut être contrôlé au moyen de robinets ou de vannes de déversoir. Diverses méthodes pour calculer l'écoulement du laminage produit par un réservoir sont examinées et comparées; la conclusion qui s'en dégage, c'est que les méthodes simples, employées normalement pour résoudre les équations différentielles, sont préférables à la méthode traditionnellement employée pour calculer le laminage produit par un réservoir et que, par conséquent, ces méthodes simples devraient remplacer la méthode traditionnelle.

INTRODUCTION

The storage equation governing the rate of change of reservoir storage volume is:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = I(t) - Q(t, S) \qquad (1)$$

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in which: S is the volume of water stored in a reservoir relative to an arbitrary datum; t is time; I is the volume rate of inflow, which is either a known function of time or known at points in time; and Q is the volume rate of outflow. To solve the equation it is necessary to relate the outflow to the storage volume, i.e. to give the form of $Q(t, S)$, usually *via* the dependence of each on the elevation of the water surface. In many discussions of the storage equation the outflow Q has been considered to depend only on S. However, where reservoir outflow is controlled by a valve or a spillway gate the outflow characteristics can be varied by human or automatic control. The outflow is then some known function of time t as well as S.

Equation (1) is a first order differential equation for S as a function of t. It can be solved numerically by any one of a number of methods, of varying complexity and accuracy, of which the most elementary are very simple indeed. The fact that the problem is merely one of solving a differential equation, and the ease with which this can be done, seem not to have been widely realised or exploited in hydrology.

The main aim of this paper is to demonstrate that the standard methods of numerical solution of differential equations work very well for reservoir routing, and that use of the traditional hydrological method has made its presentation to students and its implementation rather more difficult and time-consuming than necessary. It is suggested that the traditional method be not further implemented.

THE TRADITIONAL METHOD

The traditional method of solving equation (1), described in almost all books on hydrology, is relatively complicated. The differential equation is approximated by:

$$I(t) + I(t+\Delta) + 2S(t)/\Delta - Q[S(t)] \approx 2S(t+\Delta)/\Delta + Q[S(t+\Delta)] \tag{2}$$

where Δ is a finite step in time, and where it has been assumed that there are no externally controlled discharges, and so Q can be expressed as a function of S only. At a particular time, t, all the quantities on the left side of equation (2) can be evaluated. The equation is then a nonlinear equation for the single unknown quantity $S(t + \Delta)$, the storage volume at the next time step, which appears transcendentally on the right side. There are several methods for solving such equations, which, although not particularly difficult in principle, in practice tend to obscure with mathematical and numerical detail the underlying simplicity of reservoir routing. Textbooks at an introductory level present procedures for solving such equations by graphical methods or by inverse interpolation, while at an advanced level a number of practical difficulties may arise (Laurenson, 1986), such that in the solution of the nonlinear equation considerable attention may have to be given to pathological cases. As some of the methods are iterative, several function evaluations of the right side of equation (2) are necessary at each time step.

AN ALTERNATIVE FORM OF THE GOVERNING EQUATION

In addition to the S formulation already described, other forms of the differential equation (1) can be obtained simply. If the reservoir surface elevation, h, changes by an amount dh, in the limit as $\mathrm{d}h \rightarrow 0$ the change in storage, dS, is given by:

$$\mathrm{d}S = A(h)\,\mathrm{d}h \tag{3}$$

where $A(h)$ is the plan area of the water surface at elevation h. Substituting into equation (1), writing the outflow, Q, as a function of both t (in the case of controlled discharges) and of h (usually a simple mathematical function like $(h - y_{\text{outlet}})^{1/2}$ and/or $(h - y_{\text{crest}})^{3/2}$, where y_{outlet} is the elevation of a pipe outlet to atmosphere and y_{crest} is the elevation of the spillway crest), an equivalent form of the storage equation is obtained:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = [I(t) - Q(t,h)]/A(h) \tag{4}$$

which is a differential equation for the surface elevation itself. This equation has been presented by Chow *et al.* (1988, Section 8.3), and by Roberson *et al.* (1988, Section 10.7), but as a form supplementary to equation (1). In fact it has some advantages over that form, and this equation (4) formulation for h might generally be preferred. It makes no use of the storage volume S, which then does not have to be calculated for routing purposes.

If the traditional equation (1) form is used, then the storage volume S has to be obtained as a function of h from the integral:

$$S(h) = \int^{h} A(y)\,\mathrm{d}y \tag{5}$$

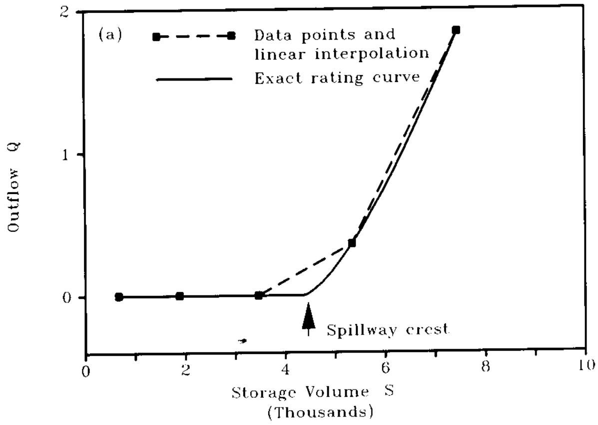
by a low order numerical approximation for various surface elevations. Then it is necessary to express S as a function of Q, usually by creating a table of pairs of corresponding values. In those cases where the discharge is controlled, this has to be repeated every time a gate or valve is adjusted. The h formulation avoids those steps. The dependence on t can be obtained by specifying a vertical gate opening or a valve characteristic as a simple function of time.

In the same way that the dependence of Q on S is usually represented by a table of pairs of values, in general the h formulation requires a similar table for A and h, to give $A(h)$, obtained from planimetric information from contour maps. However, this process is less liable to error for the reasons set out in the following argument: almost every spillway has a crest level considerably above the bottom of the reservoir, and the total operating height range of the spillway is small compared with the total depth of the reservoir. Whereas there might be a number of contour intervals used to calculate the volumes in the reservoir, there might be relatively few in the operating height range of the spillway. If the conventional S formulation is used, then the outflow Q has to be calculated as a function of S in that range, and there might be few values of S for that purpose. If this is the case, then the knowledge of Q as a function of S might have to be obtained with few data points and be correspondingly inaccurate, particularly as Q is sometimes a discontinuous and rapidly varying function of elevation. This can degrade the accuracy of the computations considerably. In the case of the h formulation this is not a difficulty, as illustrated by the following model problem.

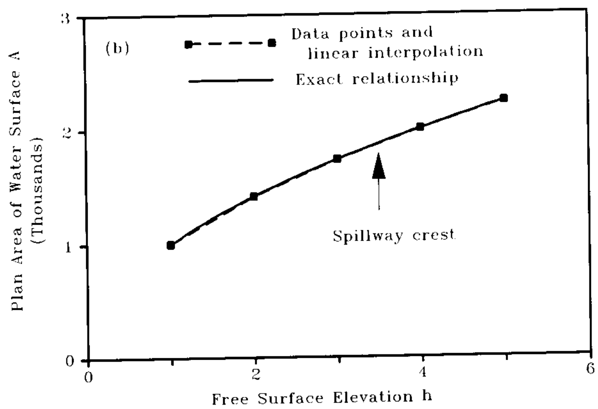
Consider a reservoir where the plan area of the water surface varies as $A = 1000\, h^{1/2}$ for typical operating levels. Integration gives the storage volume as a function of height: $S = 667\, h^{3/2}$. In a typical practical problem, contour area information is not available in the form of the precise function given here but in the form of numerical values. In this example it is presumed that they are known at intervals of 1 m between the minimum elevation of the water level at 1 m, and the maximum at 5 m. A spillway is installed with a crest at elevation 3.5 m. The rating curve is given by: $Q = 1 \times (h - 3.5)^{3/2}$ for $h \geq 3.5$ and $Q = 0$ for $h \leq 3.5$. The points making up the rating curve for Q as a function of S are shown on Fig. 1(a), as well as the linear interpolation between these points, which is often the method used in practice. It is obvious that the paucity of S information in the operating range of the spillway and the discontinuous and rapidly varying nature of Q as a function of S can cause the interpolated values of Q to be significantly in error. Numerical solution using linear interpolation would infer that the spillway starts to discharge at a value of S rather below that corresponding to the crest! Even if a higher order interpolation were used, the discontinuous nature of the function at the crest would still produce errors, with possibly negative values of Q. While it may be possible to use linear interpolation to provide an extra value of S at the spillway crest level, thus reducing the errors there, it still seems that specifying the discontinuous and rapidly varying Q as a function of the possibly sparsely known S would be a source of error.

The above situation can be compared with the h formulation in which case the details of the spillway or other outlet functions in the form of the power law expressions can be accurately evaluated for any value of h. The only interpolation for the shape of the reservoir is in the pairs of values of A and h, which generally have a smooth relationship. The values for the model problem are shown in Fig. 1(b), which shows that the accuracy of linear interpolation, and hence the evaluation of the right side of the differential equation, is rather better for the h formulation than for the S formulation. It should be noted that Q will still be a discontinuous function of h (it is zero for h below crest level); however it can be calculated accurately for all values of h.

Although great accuracy is usually not necessary in hydrological



(a) interpolating for Q as a function of S in the S formulation



(b) interpolating for A as a function of h in the h formulation

***Fig. 1** Comparison between the two formulations showing how interpolating for reservoir geometry may be an important limit to the accuracy with which the S formulation might be solved.*

computations, unnecessary inaccuracies should be avoided, and certainly simpler methods should be favoured. In view of the complications and inaccuracies associated with the S formulation as described above, it seems that the h form is to be preferred. The methods described below, however, are valid for either.

METHODS OF SOLUTION OF THE DIFFERENTIAL EQUATION

Whichever formulation is adopted, S or h, the problem is one of the numerical solution of a differential equation, for which there are many methods and much computer software available. Reference can be made to any book on numerical methods for a description of some of them. Some methods are briefly described below and their performances are compared.

The forms of the storage equation can be generalized by the expression:

$$\frac{dx}{dt} = F(t,x) \tag{6}$$

where x is the dependent variable, either S or h. In the case of equation (1), $x = S$ and $F(t, S) = I(t) - Q(t, S)$, while for equation (4), $x = h$ and $F(t, h) = [I(t) - Q(t, h)]/A(h)$. The general form in equation (6) encompasses cases where the outflow is controlled and where it is not. All the following methods can be applied whether or not control is varying the form of the outflow function with time.

To commence solution it is necessary to know the initial conditions, either the value of S or h at $t = 0$, denoted by S_0 and h_0 respectively. At later times the subscript n is used to denote the value at time $t_n = n\Delta$, and $n + 1$ at time $t_n + \Delta = t_{n+1} = (n + 1)\Delta$.

Whichever method is used, perhaps the most complicated part of solving the differential equation is that of interpolating in a table of value pairs. It is necessary to obtain $A(h)$ for arbitrary h, or $Q(t, S)$ for arbitrary S. If time steps other than the interval of the inflow hydrograph were used, it would be necessary to obtain $I(t)$ for arbitrary values of t, also from data pairs. For first and second order methods linear interpolation in a table of data pairs gives accuracy consistent with the numerical method. However, this is not the case for higher order methods, for which more accurate approximation is necessary for consistency. For reservoirs where the discharge function can have gradient discontinuities, such as where a higher spillway takes over from a lower spillway or pipe, linear interpolation would be rather more robust. Its simplicity is generally to be preferred for flood routing, where great accuracy is usually not justified.

Test problem

For the purpose of comparing the accuracy and performance of different numerical methods there are some reservoir routing problems which have analytical solutions. In particular, a case given by Yevjevich (1959, Table 3) is quite a good one for simulating real problems. The inflow hydrograph has the same shape as a typical real hydrograph, and this case might be adopted as a suitable test for developing computer programs to check on their accuracy or for comparing methods. The details are:

(a) **inflow hydrograph**:

$$I(t) = I_0 + Pt^s e^{-ft} \tag{7}$$

where P, s and f are constants defining the storm hydrograph and I_0 is a base flow added in this work for greater generality;

(b) **storage and area functions:**

$$S(h) = ah^m \qquad A(h) = amh^{m-1} \tag{8}$$

where a and m are constants; and

(c) **discharge function:**

$$Q(h) = bh^m \tag{9}$$

where b is a constant and the exponent m is the same as for the storage function. Hence, $Q = c\,S$, where $c = b/a$, a constant. This does imply that the outlet level is at the bottom of the reservoir, which might seem rather special, nevertheless this is typically experienced in detention reservoirs, and in any case this does not make the case any easier to compute.

By rewriting the differential equation with Q as the dependent variable and solving by an integrating factor method, it can be shown that this case has the analytical solution:

$$Q = I_0 + e^{-ct}\left[Q_0 - I_0 + \frac{s!cP}{(f-c)^{s+1}}\right] - \frac{s!cP}{(f-c)^{s+1}}e^{-ft}\left[1 + (f-c)t + \ldots + \frac{[(f-c)t]^s}{s!}\right] \tag{10}$$

for the case where s is an integer and where Q_0 is the initial discharge ($Q_0 = Q(t = 0)$). A solution in terms of incomplete gamma functions can be found if s is not an integer. (It should be noted that the solution corresponding to equation (10) given by Yevjevich (1959, Table 3) contains some typographical errors.)

The problem defined by equations (7) to (9) was solved by a number of numerical methods and compared with the analytical solution, equation (10), to examine their performance. The results shown below are those obtained from solving the S formulation of the differential equation (1). Numerical values used in the computations are given in Table 1. The storage details correspond roughly

***Table 1** Numerical values used in computations for Figs 2 and 3*

Quantity	Value
N = number of time steps	20
Time step Δ	300 (s)
Q_0	1.0 ($m^3\ s^{-1}$)
I_0	1.0 ($m^3\ s^{-1}$)
P	2×10^{-13} ($m^3\ s^{-1}$)
s	5
m	3/2
f	0.003 (s^{-1})
a	5000 (SI units)
b	4 (SI units)
$c = b/a$	0.0008 (s^{-1})

to those of a small detention reservoir, which has a single sharp-crested weir of length about 2.5 m, and the plan dimensions, if it were a square, would be about 100 m by 100 m if the reservoir were 2 m deep. The numerical constants associated with the inflow hydrograph were chosen by trial and error to be characteristic of those encountered in practice with a single rainstorm of short duration. The computations were not performed using equations (8) and (9) for reservoir and discharge characteristics but rather by interpolating in a table of 10 pairs of values, as would be encountered in a practical problem where values of storage or reservoir area might be known only at a finite number of points. Similarly, the inflow hydrograph was specified by 20 numerical values of $I(t)$ obtained from equation (7), and interpolation used to calculate intermediate values, where necessary. In practical problems, of course, the values would be interpolated between measured data points.

Figure 2 shows the inflow hydrograph from equation (7), the outflow hydrograph from equation (10) and various numerical solutions for the outflow hydrograph. Almost all the methods described below give results of acceptable practical accuracy. The various methods will be described and compared.

Euler's method

This is the simplest but least accurate of all methods, being of first order accuracy only. It is an explicit method, given by:

$$x_{n+1} = x_n + \Delta F(t_n, x_n) + O(\Delta^2) \tag{11}$$

and the right side of the differential equation has to be evaluated only once per time step. The Landau order symbol $O(\Delta^2)$ means that neglected terms vary like the second or higher powers of the time step Δ. In the case of equation (1), equation (11) becomes the simple expression:

$$S(t+\Delta) = S(t) + \Delta\{I(t) - Q[t, S(t)]\} + O(\Delta^2) \tag{12}$$

The evaluation of the right side of equation (12) to obtain $S(t + \Delta)$ is a much simpler process than the numerical solution of the transcendental equation (2). However, it can be seen from the test case results as shown on Fig. 2, that the accuracy of Euler's method is not particularly good. The results are in error by more than 10% at the peak. However, greater accuracy can be obtained by choosing computational steps smaller than those used to define the inflow hydrograph (when linear interpolation might be used to give intermediate values of the hydrograph).

As Euler's method is so simple, and the time interval reduction such a simple and useful artifice, this method might be the best for introducing the subject to students; and in view of the imprecise nature of much hydrological data, it might be the scheme favoured in practice.

There is another advantage to using smaller time steps. Not only would

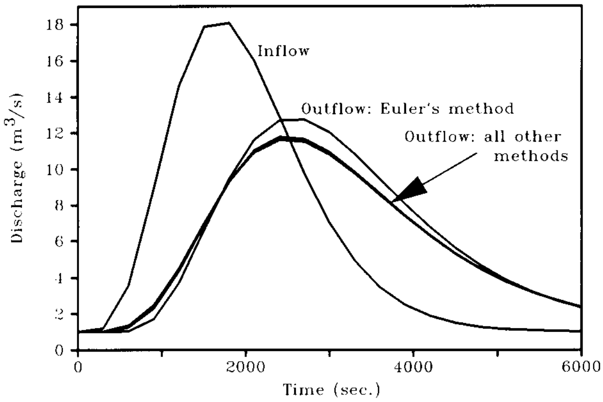


***Fig. 2** Results obtained by various computation schemes for the outflow hydrographs (theoretical inflow and outflow hydrographs obtained from equations (7) and (10), numerical values given in Table 1).*

it give rather more accurate results, but the peak of the hydrograph would be identified rather more accurately. Fig. 2 has been plotted using linear interpolation between computational points, and shows that without a more sophisticated means of interpolation, the peak of the hydrograph is underestimated; for this important quantity the accuracy of the actual computational points would be rather in vain. Using smaller computational time steps, the difference between the maximum computational point value and the actual peak would be reduced. A rather better method of determining the time and magnitude of the maximum would be to use, for example, quadratic or spline interpolation.

Figure 3 shows, for various time steps, the performance of Euler's method and various other methods for the numerical values given in Table 1, obtained by computing each time until t = 6000 s. Tests with N = 10, 20, 30 and 40 steps were made. The errors plotted are the mean errors of outflow of all of the 20 points in time where a numerical value of the inflow hydrograph was specified. It can be seen that the curve for Euler's method is parallel to the upper dashed line, corresponding on this log-log plot to the error varying like Δ, proportional to N^{-1}. Clearly the method is behaving as a first order method, such that a halving of the time step halves the error. This is in keeping with the local truncation error $O(\Delta^2)$ per time step of equation (11), as the total number of time steps is proportional to $1/\Delta$, and hence the total error at a given time is $O(\Delta)$.

Runge-Kutta second order method

The next of the methods in the hierarchy of accuracy is also known by several other names, including the "modified Euler method". It is also an explicit method, which involves only one more function evaluation per time step, and

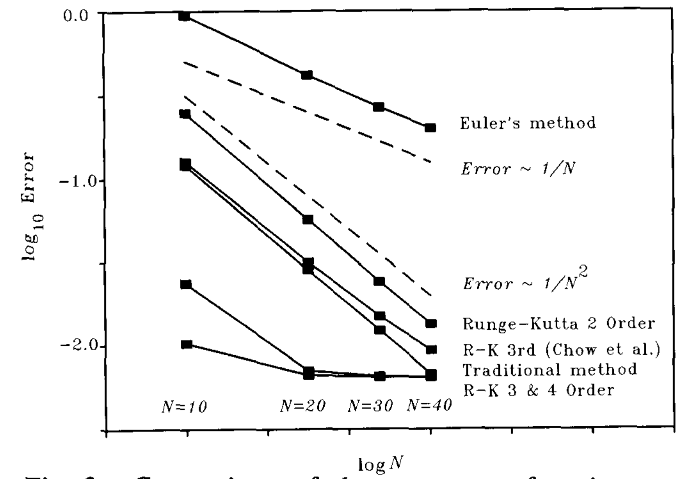


Fig. 3 Comparison of the accuracy of various schemes and dependence of accuracy on the time step used (label order as at N = 20; N = 10, 20, 30, 40 are steps of 600, 300, 200 and 150 s).

gives a scheme which is of second order accuracy. The scheme is:

$$x_{n+1} = x_n + \tfrac{1}{2}(k_1 + k_2) + O(\Delta^3) \tag{13a}$$

where:

$$k_1 = \Delta F(t_n, x_n) \tag{13b}$$

$$k_2 = \Delta F(t_n + \Delta, x_n + k_1) \tag{13c}$$

This is a second order method, as shown by the gradient of the results on Fig. 3, where the error varies like $1/N^2$. On Fig. 2 the results define the bottom edge of the thickened line containing this and all higher order results. Clearly the method is rather more accurate than Euler's method. What is convenient is that the order of accuracy is not degraded if simple linear interpolation is used to evaluate the function F at intermediate values of t and x. The results from this method on both Figs 2 and 3 were obtained using such linear interpolation.

Higher order Runge-Kutta Methods

These methods obtain the solution at $(t + \Delta)$ in terms of the solution at time t and the evaluation of $F(t, x)$ at those two times and at intermediate points. Reference can be made to most texts on numerical methods for details of the methods. The results obtained are shown on Fig. 2, where they are indistinguishable from the exact expression (10), and on Fig. 3.

Figure 3 shows an interesting phenomenon, viz. that although the higher order methods were more accurate for large time steps (small N), refining the time step had little effect on the achievable accuracy, and they did not act like third and fourth order methods. The reason for this is that the differential equation was evaluated using interpolation between discrete data points for Q as a function of S, thereby providing a limit to the achievable accuracy.

For practical calculations there seems to be little point in using methods of high order accuracy for reservoir routing. What is notable on Fig. 2 is the fact that, except for Euler's method, results obtained by all the methods described here were satisfactory. If a smaller time step were used, even Euler's method would give acceptable accuracy.

Also on Fig. 3 are shown the results of the method used by Chow *et al.* (1988) for a nominally third order Runge-Kutta scheme, but where linear interpolation in time and for the surface area function was used. It can be seen by the gradient of the results that this was an incompatible approximation and it is actually only a second order method. However, the results were slightly more accurate than the second order Runge-Kutta method (the error was halved) and this scheme might be preferred in practice.

Traditional method

This is the method based on solution of the nonlinear transcendental equation (2). Laurenson (1986) and Pilgrim (1987) have described a number of aspects of this method. Implementation of the traditional method is complicated, both at an introductory level (involving the graphical or numerical solution of a nonlinear transcendental equation at each time step), and at an advanced level, where some complicated programming is necessary to treat pathological cases. The accuracy of the traditional method for Yevjevich's test problem is shown on Fig. 3. It was, in fact, slightly more accurate than the second order Runge-Kutta method. Over the whole range of computations its error was always close to half that of the Runge-Kutta method. Both are second order.

There has been some controversy as to the validity of the traditional method based on equation (2) for the case when the discharge is controlled. Here it is shown that it is indeed a legitimate approximation to the governing differential equation. Substituting equation (1), the S formulation of the differential equation, into the implicit scheme (the Adams-Moulton second order method):

$$x_{n+1} = x_n + \Delta/2[F(t_n, x_n) + F(t_n + \Delta, x_{n+1})] + O(\Delta^3) \tag{14}$$

and regrouping terms gives:

$$I(t) + I(t+\Delta) + 2S(t)/\Delta - Q[t, S(t)] \approx 2S(t+\Delta)/\Delta + Q[t+\Delta, S(t+\Delta)] \tag{15}$$

This shows that the modification of equation (2) to allow for controlled discharge is trivial. Here, on the left, the value of Q is obtained from the storage S using the controlled discharge characteristics at time t, while on the right in the as-yet unknown term the discharge characteristics at time $t + \Delta$ are used. This is what has been done in practice, but seems to have caused some suspicion in the technical literature. Placing it in the context of differential equation theory shows that it is a consistent second order approximation. At worst it assumes that the outflow characteristics vary uniformly between the two time points. It is not based on an assumption that the discharge increases with no change in storage (*cf.* Laurenson, 1986). If a gate were fully opened or shut between two computational steps, which may well occur because the speed of gate operation is usually faster than the rate at which water levels fluctuate, then it might be necessary to vary the computational steps to describe that variation adequately.

STABILITY OF SOLUTION SCHEMES

To test the performance of the various solution methods for practical problems, they were applied to the sample problems included in several hydrology texts and reference books, nearly all of which use the traditional S formulation. In almost all cases they performed totally satisfactorily. There were two exceptions, however, where the Runge-Kutta explicit methods gave erratic results, whereas the the traditional method performed as well as for other problems. The examples were those of Hjelmfelt & Cassidy (1975, p. 122) and Pilgrim (1987, p. 141). The behaviour encountered was that of instability of the explicit numerical schemes, which was overcome by simply reducing the size of the computational time step.

It is possible to obtain theoretical criteria for the stability of solution methods for reservoir routing. Stability is usually guaranteed if the product of the step size and the magnitude of the partial derivative of the right side of the differential equation with respect to the dependent variable is limited in size. For the general scheme $\mathrm{d}x/\mathrm{d}t = F(t, x)$ (equation (6)), the second order Runge-Kutta method is stable for linear equations if:

$$\Delta \left| \frac{\partial F}{\partial x} \right| < 2 \tag{16}$$

For other schemes, the numerical value on the right side takes on other values (see, for example, Rice, 1983, Fig. 9.8). For higher order schemes the value is smaller and the criterion becomes more demanding.

Using the S formulation of the storage equation where the discharge is not controlled the criterion for stability becomes:

$$\Delta \frac{\mathrm{d}Q}{\mathrm{d}S}(S) < 2 \tag{17}$$

which can be tested before computations commence. In practice the best way of testing stability is simply to start computing, and in the rare situation that results are unstable, reduce the step size and restart computation until the instability has disappeared.

It is notable that the traditional scheme, equation (2), as used by hydrologists for many years with graphical or numerical methods of solution, is unconditionally stable. This may account for its success and continued use, despite its complexity.

There is a physical interpretation of the stability limit given by equation (17), viz. that in practice instability will tend to occur where a large change in discharge corresponds to a small change in volume. The sort of situation where this might be a problem is where an outlet pipe is placed at the bottom of a detention reservoir, where the storage volume and changes in that volume are small. This is the case presented in Pilgrim (1987). For other types of reservoirs the stability problem would be expected rarely to occur, as the gradient dQ/dS would be rather smaller. More often, the outlet level is some height above the reservoir bottom, so that when the spillway starts to operate, storage volume changes are very much greater, and stability is never a problem.

As suggested above, the easiest way of testing for stability is simply to start computing, for if a scheme is unstable this quickly manifests itself through oscillating results or numbers growing very large very quickly. If a scheme is only marginally unstable, however, this may not be noticed. It would be desirable always to solve a problem for at least two different time steps and to verify that the solutions are acceptably close to each other. In all cases the use of sufficiently smaller time steps renders the schemes stable and more accurate.

To conclude this section the question has to be posed: in view of the desirable stability properties of the traditional method, should it not be retained as the standard method of solution? The complexity of the method is a major disadvantage, yet its superior stability is not so important, for simply reducing the time step makes explicit methods stable in those rare cases where they have problems. Solving a flood routing problem by an explicit method is usually quite trivial in terms of computer resources, so that smaller time steps are not a problem. Even with smaller steps it is probable that the computing resources required are smaller than with the traditional method, which requires repeated evaluations at each time step. It is the author's opinion that the traditional method should be no longer taught or implemented.

CONCLUSIONS

Use of the traditional method for reservoir routing has obscured the fundamental simplicity of the problem, and has required considerable attention to details of the solution process. If, however, it is recognized that the procedure is one of solving a differential equation, then procedures are simpler, are more flexible and more automatic. Conclusions can be summarized as follows:

(a) Reservoir routing is simply a process of solving a differential equation, which can be done numerically by any one of a number of standard methods of varying complexity and accuracy. The most elementary methods are very simple.

(b) Using the governing equation written in terms of the reservoir surface elevation, h, has advantages over the traditional S formulation. It has been shown that use of point data to relate Q and S causes the accuracy of the governing equation and results to be highly dependent on the accuracy of the numerical determination of the storage.

(c) Various numerical methods have been examined and compared. It has been concluded that simple explicit methods are to be preferred to the traditional method. In particular, the second order Runge-Kutta method has been found to give a good combination of ease of implementation and acceptable accuracy. There is usually no need to use higher order methods.

(d) For special cases of an extreme nature, for which criteria have been presented, explicit methods become unstable. However, they can be made stable by reducing the computational time step, which also makes them more accurate. At least two different time steps should be used to provide a check on the accuracy of results.

(e) Controlled discharges: the presentation has incorporated the case where outflow from the reservoir may be varied by human or automatic control by valves or spillway gates. All the methods described can be applied to such a case, including the traditional method.

(d) It is suggested that the traditional method involving the traditional numerical solution of the transcendental equation (2) be not further implemented. Indeed, there seems to be little point in presenting it in books or lectures.

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