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THE EFFECTS OF OBSTACLES ON SURFACE LEVELS AND BOUNDARY RESISTANCE IN OPEN CHANNELS

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Abstract

Simple momentum considerations are used to show how the increase in water levels due to obstacles in a natural channel can be calculated. This requires use of a numerical method for solving nonlinear equations, which is not difficult. However it is more insightful to consider an approximate explicit theory, which shows the important quantities governing the problem, and which is accurate enough for practical purposes. It is applicable to obstacles of arbitrary extent and location, in both subcritical and supercritical flow in channels of arbitrary section. A method for the numerical refinement of this is presented, but it will usually not be necessary. The methods are compared with a theory and experimental results for a rectangular channel with vertical cylinders extending the whole depth. Then momentum-loss considerations are applied to explaining the nature of friction laws in open channels. Instead of boundary shear, momentum loss from discrete elements is used as the means of modelling resistance. It is found that the behaviour of Gauckler-Manning's law for wide channels in successfully mimicking the frictional behaviour over a range of depths can be explained, but it does not yet provide a comprehensive theory for general cross-sections.

Keywords: afflux, canals, debris, Gauckler, inundation, Manning, momentum, open channels, rivers, roughness, surface level.

INTRODUCTION

An estimate of the increase in surface levels due to obstacles in a channel can be done using simple momentum considerations which require the numerical solution of a nonlinear equation. Henderson (1966, p72 & p264) outlined the important considerations and procedures, while Montes (1998, #8-10) described empirical work by Rehbock and Yarnell, and used the momentum approach to produce formulae for the backwater effect due to bridge piers extending the whole depth in rectangular waterways. Such calculations are becoming more important for natural waterways, in the installation of large woody debris for environmental purposes. Gippel, O'Neill, Finlayson, and Schnatz (1996) presented a summary of research in this area. Calculations of the effects of momentum loss were based on the theory presented by Ranga Raju, Rana, Asawa and Pillai (1983), which has some limitations in that the theory is for a rectangular channel, for subcritical flow, and the obstacles are assumed to be subject to the mean velocity in the channel. Also Shields and Gippel (1995) developed a method for estimating the effects of debris, bed material, bars, and bends on flow resistance in rivers. In this paper more general methods are developed for arbitrary cross-sections, and subcritical and supercritical flow. A simple explicit approximation is obtained, which can be refined if necessary. Finally, consideration is given to the nature of roughness in channels and how it might be described by considerations of momentum loss from a number of discrete elements. This is found to give an explanation for the success of the Gauckler-Manning equation for wide streams at least.

MOMENTUM THEORY FOR ARBITRARY SECTION

Consider an obstacle or obstacles in a steady open channel flow, such as the piers of a bridge, blocks on the bed, vanes, the bars of a trash-rack, or individual roughness elements such as vegetation or bed material. The drag force D on the obstacle can be calculated in terms of fluid density ρ, drag coefficient C_d, local mean horizontal fluid velocity impinging on the object U, and the area of the object measured transverse to the flow, a. The expression from conventional fluid mechanics is $D = 1/2 \times \rho C_d U^2 a$. Now consider steady flow in the channel. The conservation of momentum theorem can be written in terms of the difference in the momentum flux between a section upstream of the obstacle at station 1 and one at 3 further downstream, such that $D = M_1 - M_3$, where M is the momentum flux at a section $M = \rho(gAh_c + \beta Q^2 / A)$, where g is gravitational acceleration, A is cross-sectional area of the channel, h_c is the depth of the centroid of the section below the surface (such that Ah_c is the first moment of area of the section about a transverse axis at the water level), β is a Boussinesq coefficient allowing for the non-uniformity of velocity over the section, $\beta Q^2 / A = \int_A u^2 \, dA$, where u is the horizontal velocity component, and Q is the discharge. Substituting into the momentum theorem:

$$\tfrac{1}{2}\rho C_d U^2 a = \rho\left(gAh_c + \beta\frac{Q^2}{A}\right)_1 - \rho\left(gAh_c + \beta\frac{Q^2}{A}\right)_3 . \tag{1}$$

The velocity U on the drag-producing element is expressed in terms of the mean upstream velocity Q/A_1, such that $U^2 = \gamma Q^2 / A_1^2$, where γ is a coefficient which allows for the fact that the velocity which impinges on the object is not necessarily equal to the mean velocity in the flow. For a small object near the bed, γ could be quite small; for an object near the surface it would be greater than one; for objects of a vertical scale that of the whole depth, $\gamma \approx 1$. Substituting and dividing equation (1) by ρg gives

$$\tfrac{1}{2}\gamma C_d \frac{Q^2}{gA_1^2} a = \left(Ah_c + \beta\frac{Q^2}{gA}\right)_1 - \left(Ah_c + \beta\frac{Q^2}{gA}\right)_3 . \tag{2}$$

A typical problem is where the downstream water level is given, such that the flow is sub-critical, the control is downstream, and it is necessary to know by how much the water level would be raised upstream if an obstacle were placed in the flow. A contemporary example is the effect on flood levels if large woody debris is placed in the stream. As both A_1 and h_{c1} are functions of the surface elevation, the solution is given by solving this nonlinear equation for that surface elevation. This is not a complicated procedure, however it is interesting to consider an approximate procedure that allows an explicit solution which provides a simple answer and insight into the problem.

AN EXPLICIT APPROXIMATION

An approximate method is considered for obtaining the rise in water level for relatively small channel obstructions. We consider a linear approximation to equation (2), which can be interpreted as a local straight-line approximation to the momentum diagram for a small reduction in momentum, which yields an explicit formula for the effect of obstacles on a flow. Consider a small change of surface elevation $\Delta\eta$ between station 1 upstream and station 3 downstream of an obstacle which does not change the momentum flux much. The undisturbed flow is assumed to be uniform, where in common with much simple open channel hydraulics it is assumed that locally friction and gravity are approximately in balance such that both are neglected. That this need not necessarily the case provides support for the approximate nature of this analysis. If the surface width is B, then it is easily shown that the elemental change in cross-sectional area is $B\Delta\eta$ such that

$$A_3 = A_1 + B\Delta\eta + O((\Delta\eta)^2),$$

where the Landau order symbol $O(\ldots)$ means that neglected terms vary at most like those shown. The first moment of area of the downstream section about a transverse axis at the water level is

$$(Ah_c)_3 = A_1(h_{c1} + \Delta\eta) + B\Delta\eta \times \tfrac{1}{2}\Delta\eta = A_1 h_{c1} + A_1\Delta\eta + O\left((\Delta\eta)^2\right),$$

and so equation (2) becomes

$$\begin{aligned} \tfrac{1}{2}\gamma C_d \frac{Q^2}{gA_1^2} a &= -A_1\Delta\eta + \beta\frac{Q^2}{gA_1} - \beta\frac{Q^2}{g(A_1 + B\Delta\eta)} + O\left((\Delta\eta)^2\right) \\ &= -A_1\Delta\eta + \beta\frac{Q^2 B}{gA_1^2}\Delta\eta + O\left((\Delta\eta)^2\right) \end{aligned},$$

where a power series expansion in $\Delta\eta$ has been used. Neglecting the higher-order terms, this is now a linear equation that can be solved to give an expression for the change in surface elevation for a force applied to a flow:

$$\frac{\Delta\eta}{A_1/B_1} = \frac{\tfrac{1}{2}\gamma C_d F_1^2 \dfrac{a}{A_1}}{\beta F_1^2 - 1},$$

where $F_1^2 = Q^2 B_1 / gA_1^3$ is the square of the Froude number at section 1. It does not matter that for subcritical flow the conditions at point 1 might not be known: within the linearising approximation, either of the values at 1 or 3 can be used in this expression, and so, generalising by dropping the subscripts altogether,

$$\frac{\Delta\eta}{A/B} = \tfrac{1}{2}\gamma C_d \frac{a}{A}\frac{F^2}{\beta F^2 - 1}. \tag{3}$$

Thus, the relative change of depth (change of depth divided by mean depth) is proportional to a coefficient expressing the mean velocity on the obstacle to the mean velocity in the channel, the drag coefficient, the fractional area of the blockage, and a term which is a function of the square of the Froude number. For subcritical flow the denominator is negative, and hence so is $\Delta\eta$, so that the surface drops after an obstacle, as we expect, and as can be seen when we solve the problem exactly using the momentum diagram. If upstream flow is supercritical, the surface rises. Clearly, if the flow is near critical, $\beta F^2 \approx 1$, the change in depth will be large (the gradient on the momentum diagram is vertical), when the theory, and all hydraulic theory, has limited validity. Whereas equation (3) provides a dimensionless insight, in practice it is simpler to evaluate it in terms of physical quantities:

$$\Delta\eta = \frac{u^2}{2g}\frac{C_d}{\beta F^2 - 1}\frac{a}{A}, \tag{4}$$

where u is the actual fluid velocity at the obstacle. This is similar to an equation presented by Montes (1998, p396) for the effects of bridge piers in a rectangular channel, but where a series expansion has been used, obscuring the fact that as flow approaches critical, effects become large and the theory breaks down. Usually in river engineering the quantities in equation (4) are at best approximately known, as the following example illustrates. Although the calculation is approximate only, it was important in a practical study to demonstrate to riparian landholders that the small effects on river levels of large woody debris installed for environmental purposes.

Example: *Calculate the effect on river levels of a straight habitat log of diameter 600mm and length 10m installed on the bed of a stream at 30º to the stream of width 100m and depth 6m. The maximum mean daily discharge is 3500* m^3s^{-1}.

The projected area of the log in the direction of flow is $10 \times 0.6 \times \sin 30 = 3\,\mathrm{m}^2$, *hence* $a/A = 3/600$. *The river is wide enough that it can be assumed to be rectangular, in the absence of any other information. The Froude number is estimated approximately using* $g \approx 10\,\mathrm{ms}^{-2}$ *as* $F = (3500/600)/\sqrt{10\times 6} \approx 0.75$. *The cross-section of the log in the direction of flow of the water is an elliptical cylinder with ratio of axes* $1/\sin 30 : 1 = 2:1$, *for which the drag coefficient for turbulent flow is* $C_d = 0.2$ *(White, 1986, Table 7.2). To estimate the velocities a power law with* $u = U_{\max} \times (z/d)^{1/7}$ *is used. The value of* β *can be calculated from this, which is so close to one that we can use* $\beta = 1$. *Requiring that the integral of the velocity give the required discharge gives* $U_{\max} = 6.7\,\mathrm{ms}^{-1}$, *and so the velocity at a height of half the log diameter becomes* $4.3\,\mathrm{ms}^{-1}$. *Substituting into equation (4) gives* $\Delta\eta = -0.002\,\mathrm{m}$, *or a drop of* $2\,\mathrm{mm}$. *Locally of course, it will be more, but as the momentum defect diffuses through the water, this will be the overall effect on the stream.*

HIGHER-ORDER SOLUTIONS

Now procedures for refinement of the solution will be developed. The problem is really just the numerical solution of a nonlinear equation, for which the previous solution was not really necessary. What it provided, however, was a simple theory which revealed the dominant features governing the problem. The author initially developed the above theory to second order to obtain a more accurate version. In fact, the resulting second-order expression was very long indeed, and it was decided not to present that approach. Instead, a well-known numerical method is applied. Newton's method (see any introductory book on numerical methods) for the solution of transcendental equations is, if $f(\eta_1) = 0$ is the equation to be solved for the quantity η_1, and if $\eta_1^{(n)}$ is an estimate of the solution, then a better approximation is:

$$\eta_1^{(n+1)} = \eta_1^{(n)} - \frac{f(\eta_1^{(n)})}{\frac{df}{d\eta_1}(\eta_1^{(n)})}. \tag{5}$$

Using equation (2), such that

$$f(\eta_1) = \left(Ah_c + \beta\frac{Q^2}{gA}\right)_1 - \left(Ah_c + \beta\frac{Q^2}{gA}\right)_2 - \tfrac{1}{2}\gamma C_d \frac{Q^2}{gA_1^2}a = 0, \tag{6}$$

then differentiating with respect to η_1 and using the results that $d(Ah_c)/d\eta = A$, $dA/d\eta = B$, both already used above, and $da/d\eta = b$, the width of the obstacle at the surface (which might be zero if the obstacle is submerged), the result is obtained:

$$\frac{df}{d\eta_1} = A_1\left(1-\beta F_1^2\right) + C_d F_1^2\left(a(\eta_1) - \frac{A_1}{B_1} b(\eta_1)\right), \tag{7}$$

and in both expressions, any quantity with a subscript 1 is evaluated with surface elevation η_1. The improved estimate of this quantity follows, using Newton's method, equation (5). This can be repeated until convergence occurs. Below it will be shown that a single pass gave four-figure accuracy in the change of depth. In terms of the actual depth, the linear approximation (3) gave three-figure accuracy.

VERTICAL CYLINDERS IN A RECTANGULAR CHANNEL

Ranga Raju *et al.* (1983) considered the special case of a rectangular stream with equispaced vertical cylinders extending the whole depth, implicitly assuming $\gamma = 1$ and also that $\beta = 1$. Considering the stream to have width *B*, with a vertical cylinder of diameter *b*, and substituting into equation (2) using $a = bh_1$, and at both sections 1 and 3 $A = Bh$ and $h_c = h/2$, they presented a quadratic equation and presented one solution which in the nomenclature of this work can be written

$$\frac{\Delta\eta}{h_3} = \tfrac{1}{3}\left(1 - F_3^2 - \sqrt{\left(1-F_3^2\right)^2 + 3C_d \tfrac{b}{B} F_3^2}\right). \tag{8}$$

That expression is, however, valid only for subcritical flow – in supercritical flow the sign of the square root term must be changed. In the spirit of the more general study above, it is interesting to obtain a lower-order approximation, and to expand as a power series in terms of the drag term, giving the linear solution

$$\frac{\Delta\eta}{h_3} = -\frac{C_d \tfrac{b}{B} F_3^2}{2(1-F_3^2)}, \tag{9}$$

which is valid for both subcritical and supercritical flow, provided the momentum loss is not too large. It is easily shown that, equation (3) reduces to this equation for a rectangular channel and $\gamma = \beta = 1$ (with the different sign convention noted above). It is interesting that in equation (8) the real role of Froude number is not made clear, because the leading terms in which it appears tend to cancel.

Figure 1 shows the experimental results from Ranga Raju *et al.* for the afflux, the increase in water level upstream due to the presence of a number of cylinders in an otherwise uniform flow. They indirectly measured the force on a cylinder and the afflux across the cylinders, and applied their theory, equation (8). In this present work two more sets of results were calculated. First, the linear approximation developed here, equation (3) (or, (9)) was used, with results shown by diamonds on the figure. Then a single pass of Newton's method (equations (5), (6), and (7)) was applied. Results are shown by squares, as well as the second-order approximation of Ranga Raju *et al* shown by crosses. The quadratic approximation closely agrees with the accurate Newton's method solutions, which had converged to four-figure accuracy after one step. For small disturbances to the flow, and hence small change of surface elevation, the simple explicit linear expression also was accurate. However, for large disturbances to the flow (when the momentum loss was as much as 10% of the momentum flux in the channel), it was not so accurate. It is clear, however, that it is sufficiently accurate for practical situations where the details of the flow distribution, the drag coefficient, and possibly even the geometry of the obstacle in the flow might be poorly known. It should be remembered that the quantities shown above are the *changes* in surface elevation which shows differences between the methods. When the actual surface elevations were considered, even the linear method was accurate to within 2mm in a total depth of 100-200mm, so that in practical problems it seems to be quite accurate enough. While the expression of Ranga Raju *et al.* is rather simpler to apply than the Newton's method here, it should be remembered that it only applies to rectangular channels where the obstacles extend throughout the depth of flow.

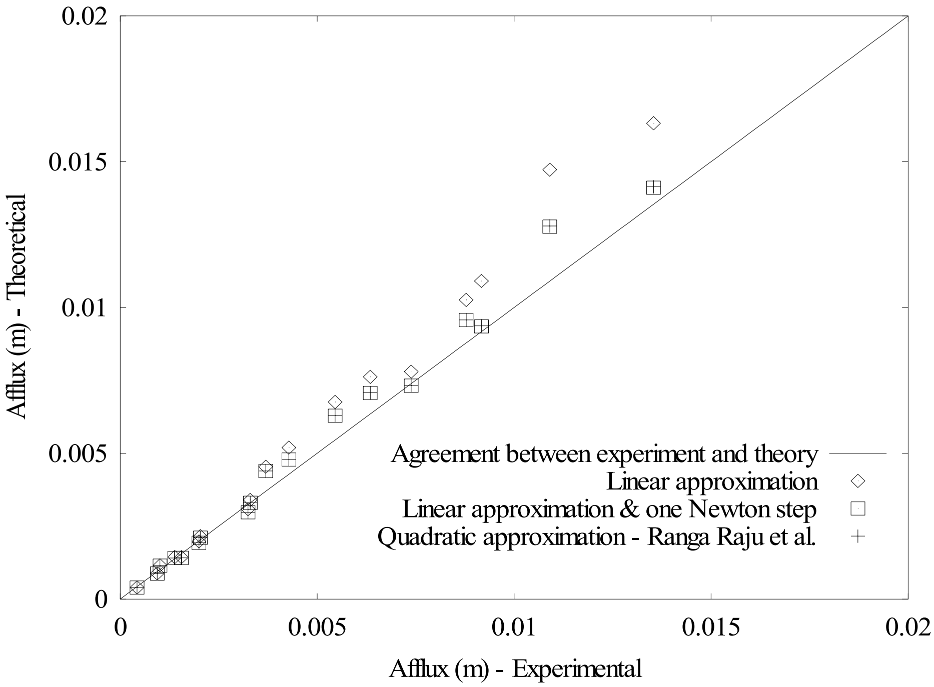

Figure 1. Change of surface elevation (afflux) across obstacle

THE NATURE OF BOUNDARY ROUGHNESS IN A CHANNEL

Considering a slice of water in an open channel carrying steady uniform flow it can be shown that the net horizontal component of force due to gravity is $\rho g A S_0 \Delta x$, where S_0 is the bed slope and Δx the thickness of the slice. Suppose that this force is resisted by the combined drag force on a number of discrete roughness elements and/or vegetation elements, so that the drag force can be written $\frac{1}{2}\rho \sum C_d u^2 a$, where the summation is over all the drag-producing elements in the volume, almost all probably around the boundary. In this it will be assumed that the channel is not very steep so that all considerations of velocity components and force components being in the horizontal x direction or along the bed of the channel can be glossed over. Equating the two forces gives

$$gAS_0\Delta x = \tfrac{1}{2}\sum C_d u^2 a ,$$

and, as above, assuming that on a drag element $u^2 = \gamma \times Q^2 / A^2$,

$$gAS_0\Delta x = \frac{1}{2}\frac{Q^2}{A^2}\sum C_d \gamma a . \qquad (10)$$

Now it is assumed that the sum over all the drag-producing elements is proportional to the wetted perimeter, such that $\frac{1}{2}\sum C_d \gamma a = \overline{C_d}\, P \Delta x$, and it is trivial to show that $\overline{C_d} = f/8$, where f is the Darcy friction factor. Substituting into equation (10) gives the equation

$$Q^2 = \frac{8g}{f}\frac{A^3}{P} S_0 ,$$

which is simply Chézy's law as it is often presented in textbooks, derived on the basis of shear stresses rather than drag forces as here. It is trivial to show that

$$\text{Chézy's coefficient } C=\sqrt{\frac{8g}{f}} \text{ and Manning's coefficient } n=\left(\frac{A}{P}\right)^{1/6}\sqrt{\frac{f}{8g}}. \tag{11}$$

Now the behaviour of f is examined. Consider a stream modelled by roughness elements on the bottom of a stream, which we will assume to be wide so that the problem is two-dimensional, and the total area of the elements does not increase as the depth increases, and neither does the wetted perimeter. In this case the flow over each of the elements should be the same such that γ is constant and if they are the same shape such that C_d is constant, then

$$f=\frac{4\sum C_d\gamma a}{P\Delta x}=4C_d\gamma\times\frac{\sum a}{P\Delta x}=4C_d\gamma\times\frac{\text{Total drag - producing area transverse to flow}}{\text{Wetted area of boundary}}. \tag{12}$$

Assuming that the drag coefficient on roughness elements is constant, the coefficient γ is the only quantity which varies with stream depth, as it is given by

$$\gamma=(\text{Velocity at roughness elements / Mean velocity in channel})^2. \tag{13}$$

Suppose that the roughness elements extend a height k into the stream, which is of total depth h. Then the velocity distribution can be written (see, for example, Montes, 1998, p85) in terms of u_* the shear velocity, the von Kármán constant $\kappa=0.4$, and the vertical co-ordinate z:

$$u=\frac{u_*}{\kappa}\ln\frac{z}{k}+8.5=\frac{u_*}{\kappa}\ln\frac{30z}{k}. \tag{14}$$

The mean velocity in the flow is obtained by integrating equation (14), such that

$$\bar{u}=\frac{u_*}{\kappa}\left(\ln\frac{30h}{k}-1\right). \tag{15}$$

Substituting $z=k$ into equation (14) to give the velocity at the roughness elements, and then into equation (13) gives

$$\gamma=\left(\frac{\ln 30}{\ln\frac{30h}{k}-1}\right)^2, \tag{16}$$

showing that the velocity at the roughness elements is a function of relative roughness. From equations (11) and (12) then, it is possible to show the variation of the Chézy and Manning coefficients with relative roughness by normalising to the value at $h/k=10$:

$$\frac{C}{C_{h/k=10}}=\sqrt{\frac{\gamma_{h/k=10}}{\gamma}}=\frac{\ln\frac{30h}{k}-1}{\ln 300-1} \quad\text{and}\quad \frac{n}{n_{h/k=10}}=\frac{\ln 300-1}{\ln\frac{30h}{k}-1}\left(\frac{h/k}{10}\right)^{1/6}. \tag{17}$$

Results are shown on Figure 2, and it is obvious how the Chézy coefficient varies markedly with relative depth, while Manning's coefficient, based on a logarithmic velocity profile, varies but little. The calculations were repeated by calculating the variation of Manning's coefficient for power law velocity profiles, and the results are shown. For a velocity-law exponent of 1/6, Manning's coefficient does not vary at all, clearly related to Strickler's relationship (Montes, 1998, p103). Unfortunately this approach cannot immediately be applied to more general cross-sections, to develop a general theory of boundary roughness, as the applicability of the logarithmic velocity law is not so sure, and the shear stress on the boundary will vary.

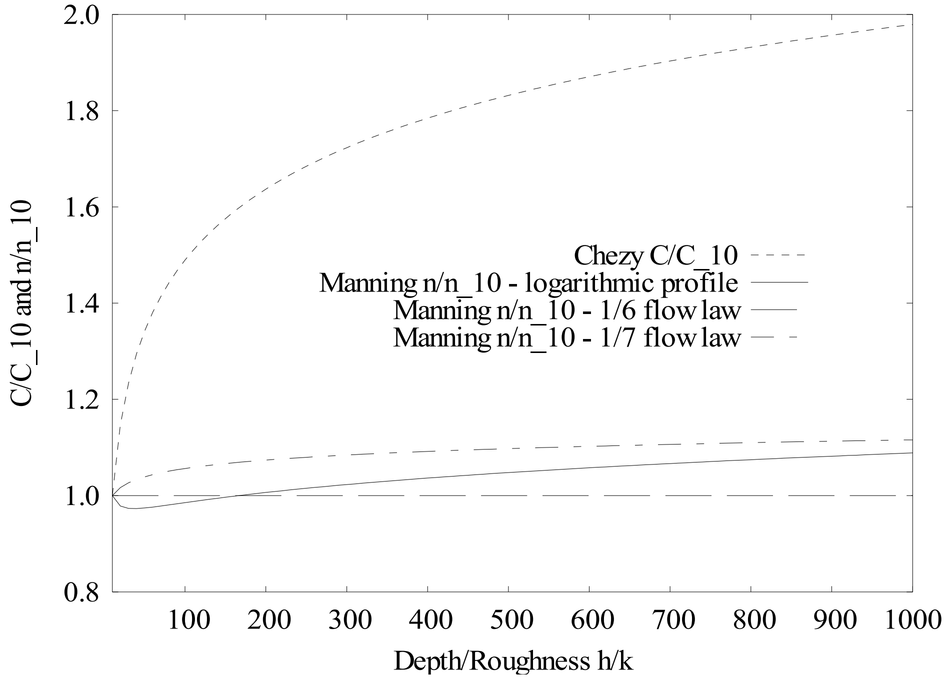

Figure 2. Variation of roughness coefficients with relative depth of flow

CONCLUSIONS

The relationship between the drag force on an obstacle in a channel flow and the change in water level across the obstacle has been considered. A simple approximate theory has been developed for obstacles of arbitrary extent and location in channels of arbitrary cross-section and for both subcritical and supercritical flow. That theory can be refined by a numerical procedure, however, it is accurate enough for most practical situations, and shows the important parameters of the problem. Amongst other applications, it can be used for calculating increased water levels due to the provision of woody debris in rivers.

Then the problem of the nature of boundary roughness in open channels is considered. Shear stresses on the boundary are modelled by the drag on roughness elements. Then by using simple two-dimensional flow laws for a wide stream it is shown how the drag varies with depth of flow, and this is seen to be mimicked quite well by the Gauckler-Manning equation with constant roughness coefficient, going a little distance to theoretically justifying its use.

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