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Rating Curves: Part 1 – Correction for Surface Slope

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Abstract: Rating curves are the means by which readings of surface elevation at gauging stations on rivers are converted to discharges. Their implicit assumption of a unique relationship between stage and discharge overlooks some of the hydraulics of the problem by ignoring the effects of surface slope. This paper describes a method for allowing for possible variable slope by incorporating time derivatives of the stage readings, providing an extension to the Jones method. In many cases the correction is indeed not necessary. To this end, formulae are provided for calculating when such corrections should be made. An even sounder procedure would be, however, always to measure the slope of the surface at stations and, rather than developing and using rating curves (stage-discharge curves), stage-conveyance curves would be generated and used so that the method used to calculate flows would be based on the actual hydraulics of each situation.

Keywords:. backwater, canal, discharge, flood, flow, gauging, hydrograph, rating curve, river, routing.

1 INTRODUCTION

Chester (1986) noted that at the AWRC Workshop on Surface Water Data in Canberra in 1983, J.A.H. Brown, the surface water resource consultant for the Commonwealth Government's 'Water 2000' study, called for the need to examine and "rationalise the whole question of discharge rating curves" as it "has received relatively little attention in technical literature" and "is covered in a general fashion in the stream gauging manuals". Since 1983 there has been little progress along those suggested directions, despite the importance of determining the actual state of a river – and irrigation canals, both under extreme events and in daily hydrologic practice. To address some of problems identified by the water industry and by researchers, in September 1997 the Cooperative Research Centre for Catchment Hydrology began a research program "Hydraulic Derivation of Stream rating Curves", which had as its main objectives:

- To improve current methods of converting measured water levels to flow rates, especially for high flows; and
- Thereby to improve the reliability of flood estimates.

The Project Leader was Assoc. Prof. R. J. Keller. The author of this paper and the companion paper (Fenton, 2001) was employed as a Research Fellow to conduct the investigation. The main outcome of the project was a report (Fenton and Keller, 2001) published by the Cooperative Research Centre for Catchment Hydrology. In the course of conducting the investigation which led to the report, a substantial review of the nature of the long wave equations governing the motion of flows and floods in rivers and canal was conducted, and a number of theoretical results obtained, which were described in the Appendices of the report. This paper and its companion extract some of the more practical results in that report.

Almost universally the routine measurement of the state of a river is that of the *stage*, the surface elevation at a gauging station, usually relative to an arbitrary local datum. While surface elevation is an important quantity in determining the danger of flooding, another important quantity is the actual flow rate past the gauging station. Accurate knowledge of this instantaneous discharge – and its time integral, the total volume of flow – is crucial to many hydraulic investigations and to practical operations of a river and its chief environmental and commercial resource, its water. Examples include decisions on the allocation of water resources, the design of reservoirs and their associated spillways, the calibration of models, and the interaction with other computational components of a network.

The traditional way in which volume flow is inferred is for a *rating curve* to be derived for a particular gauging station, which is a relationship between the stage measured and the flow passing that point. The measurement of flow is done at convenient times by traditional hydrologic means, with a current meter measuring the flow velocity at enough points over the river cross section so that the volume of flow can be obtained for that particular stage, measured at the same time. By taking such measurements for a number of different stages and corresponding discharges over a period of time, a number of points can be plotted on a *stage-discharge diagram*, and a curve drawn through those points, giving what is hoped to be a unique relationship between stage and flow, the rating curve. This is then used in the future so that when stage is routinely measured, it is assumed that the corresponding discharge can be obtained from that curve.

There are many problems associated with the use of a rating curve, of which some are:

- The assumption of a unique relationship between stage and discharge is, in general, not always warranted, although in many cases it might be accurate enough for practical purposes. An additional determinant is the *slope* of the free surface, which can vary over a flood event, so that the flow on the rising limb of a flood is higher for a given stage than for the falling limb when the surface slope is smaller.
- Discharge is rarely measured during a flood, and the quality of data at the high flow end of the curve might be quite poor.
- It is usually some sort of line of best fit through a sample made up of a number of points – sometimes extrapolated for higher stages.
- It has to describe a range of variation from no flow through small but typical flows to very large extreme flood events.

This paper addresses the first of these problems, while the companion paper (Fenton, 2001) considers aspects raised by the last two.

2 THE HYDRAULICS OF A GAUGING STATION

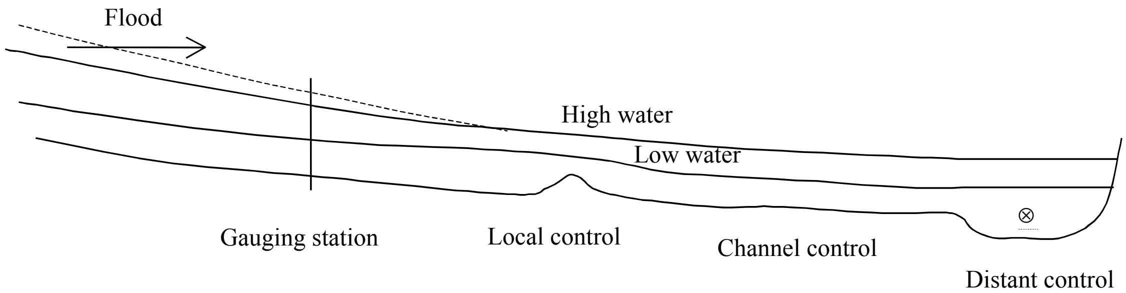

Figure 1. Typical Gauging Station, Showing Controls At Different Water Levels
and a Flood Moving Downstream

A typical set-up of a gauging station where the water level is regularly measured is given in Figure 1, which shows a longitudinal section of a stream. Downstream of the gauging station is usually some sort of fixed control which may be some local topography such as a rock ledge, which means that for relatively small flows there is a relationship between the head over the control and the discharge which passes. This will control the flow for small flows. For larger flows the effect of the fixed control is to become unimportant, to "drown out", and for some other part of the stream to control the flow. If the downstream channel length is long enough before encountering another local control or waterway, for a mild slope the section of channel downstream will itself become the control, where the control is due to friction in the channel, giving a relationship between the flow and the slope in the channel, the stage, channel geometry, and roughness. Finally, control might be due to a larger river downstream shown as a distant control in the figure, or even the sea. There may be more intermediate controls too, but in practice, the precise natures of the controls are usually unknown. Although the factors affecting the stage and discharge at a gauging station shown in Figure 1 seem complicated, the underlying processes are capable of quite simple description.

2.1 Low Flow

If the flow is small then the gauging station is likely to be under the influence of a local control downstream. There might be something like a rock ledge or an installed weir providing a unique relationship between stage and flow at that point. Between that point and the gauging station upstream, unless they are very close together, the governing equations for open channel flow will apply.

In Fenton and Keller (2001) the hydraulics of a downstream control and a gauging station are considered, and the problem is solved assuming that the channel between the two is prismatic, obtaining an analytic solution to the gradually varied flow equations. It is shown that *in the low flow limit* at the gauging station,

$$h = h_{\mathrm{csf}} + H_c(Q) + \text{Terms of order } Q^2 , \tag{1}$$

where h is the depth at the gauging station, h_{csf} is the cease-to-flow depth there, $H_c(Q)$ is the head at the control, shown as a function of discharge Q. Generally, for sharp and broad-crested weirs this will vary something like $Q^{2/3}$ for a rectangular cross-section, $Q^{1/2}$ for a parabolic cross-section, or $Q^{0.4}$ for a triangular cross section. This behaviour is considered in some detail in Fenton (2001). Of course, if the control is a natural feature the behaviour may be more complicated, but this gives an idea of the behaviour. The neglected terms of order Q^2 shown give the head loss between the gauging station and the control, and follow simply from application of Manning's or Chézy's laws, where the surface slope, giving the additional elevation at the upstream gauging station, is proportional to Q^2. For sufficiently low flow these will be negligible, and the reach between gauging station and control is essentially a reservoir with flow through it, where dynamical effects on the surface are negligible and the surface is horizontal with elevation given by the control. Hence the stage-discharge relationship is given by that of the control. This makes matters rather difficult in deriving a rating curve theoretically, as the complicated geometry of natural controls makes a theoretical determination usually not possible. This is considered in more detail in Fenton (2001), which considers means of representing and approximating rating curves.

This shows that there is a relationship between stage and discharge at the gauging station when it is under the effects of the local control. When the control is washed out for larger flows control is effectively by stream control, which is now considered, when the *slope* of the surface becomes important.

2.2 Intermediate and High Flows

In a typical stream, where all wave motion is of relatively long time and space scales, the governing equations are the long wave equations, which are a pair of partial differential equations for the stage and the discharge at all points of the channel in terms of time and distance along the channel. One is a mass conservation equation, the other a momentum equation. Under the conditions typical of most flows and floods in natural waterways, however, the flow is sufficiently slow that the equations can be simplified considerably. It can be shown that most terms in the momentum equation are of a relative magnitude given by the square of the Froude number, which is U^2/gd, where U is the fluid velocity, g is the gravitational acceleration, and d is the mean depth of the waterway. In most rivers, even in flood, the square of the Froude number is small. For example, a flow of 1 m/s with a depth of 2m has $F^2 \approx 0.05$. Under these circumstances, a surprisingly good approximation to the momentum equation of motion for flow in a waterway is the simple equation:

$$\frac{\partial \eta}{\partial x} + S_f = 0 , \tag{2}$$

where η is the surface elevation, x is distance along the waterway and S_f is the friction slope. Usual practice is to use an empirical friction law for the friction slope in terms of a conveyance function K, so that it is written

$$S_f = \frac{Q^2}{K^2} , \tag{3}$$

in which Q is the instantaneous discharge, and where the dependence of K on stage at a section may be determined empirically, or by a standard friction law, such as Manning's or Chézy's law:

$$\text{Manning's law: } K = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}} \text{ or Chézy: } K = C\frac{A^{3/2}}{P^{1/2}} , \tag{4}$$

where n and C are Manning's and Chézy's coefficients respectively, while A is cross-sectional area and P is wetted perimeter, which are both functions of depth and x, as the cross-section usually changes along the stream. In most hydrographic situations K would be better determined by direct measurements rather than by these formulae as they are approximate only and the roughness coefficients are usually poorly known.

Even though Manning's and Chézy's laws were originally intended for flow which is both steady (unchanging in time) and uniform (unchanging along the waterway), they have been widely accepted as the governing friction equations in more generally unsteady and non-uniform flows. Hence, substituting (3) into (2) gives an expression for the discharge, where the functional dependence of each variable is now shown:

$$Q(t) = K(\eta(t))\sqrt{S_\eta(t)}\,, \tag{5}$$

where the symbol $S_\eta = -\partial\eta/\partial x$ has been introduced for the slope of the free surface, positive in the downstream direction, in the same way that the symbol S_f has been used for the friction slope. This gives us an expression for the discharge at a point and how it might vary with time. Provided (a) the stage and the dependence of conveyance K on stage at a point are known from either measurement or Manning's or Chézy's laws, and (b) the slope of the surface, (5) gives a formula for calculating the discharge Q which is as accurate as is reasonable to be expected in river hydraulics. It shows how the discharge actually depends on both the stage and the surface slope.

3 INCORPORATING HYDRAULIC THEORY: STAGE-CONVEYANCE CURVES

Traditional hydrography assumes that discharge depends on stage alone. In many situations this might be a good approximation to reality, where the slope does not vary much. If the slope does vary under different backwater conditions or during a flood, then a better hydrographical procedure would be to gauge the flow *and to measure the surface slope* S_η, thereby enabling a particular value of K to be calculated for that stage. If this were done over time for a number of different stages, then a *stage-conveyance* relationship could be developed which should then hold whether or not the slope is varying. Subsequently, in day-to-day operations, if the stage and the surface slope were measured, then the discharge calculated from (5) should be quite accurate, within the relatively mild assumptions made in its derivation.

If hydrography had followed the path described above, of routinely measuring surface slope and using a stage-conveyance relationship, the science would have been more satisfactory. Effects due to the changing of downstream controls with time, downstream tailwater conditions, and unsteadiness in floods would have been automatically incorporated, both at the time of determining the relationship and subsequently in daily operational practice. However, for the most part slope has not been measured, and hydrographical practice has been to use rating curves instead. The assumption behind the concept of a *discharge-conveyance* relationship or *rating curve* is that the slope at a station is a unique function of stage only, so that the discharge is a unique function of stage $Q_\mathrm{r}(\eta)$ where the subscript r indicates the rated discharge. Instead of the empirical-rational expression (5), traditional practice is to calculate discharge from the equation

$$Q(t) = Q_\mathrm{r}(\eta(t))\,, \tag{6}$$

thereby ignoring any effects that downstream backwater and unsteadiness might have, as well as the possible changing of a downstream control with time. In comparison, (5), based on a convenient empirical approximation to the real hydraulics of the river, contains the essential nature of what is going on in the stream – for intermediate and large flows. It shows that, although the conveyance might be a unique function of stage that it is possible to determine by measurement, because the surface slope will in general vary throughout different flood events and downstream conditions, discharge in general does not depend on stage alone.

The above argument suggests that ideally the concept of a stage-discharge relationship be done away with, and replaced by a stage-conveyance relationship. Of course in many, even most, situations it might well be that the surface slope at a gauging station does vary but little throughout all conditions, in which case the concept of a stage-discharge relationship would be accurate. Below this is quantified and criteria obtained. In most situations it is indeed the case that there is little deviation of results from a unique stage-discharge relationship.

Now a means is provided for correcting for variable slope when that has not been measured.

4 CORRECTING FOR UNSTEADY EFFECTS IN OBTAINING DISCHARGE FROM STAGE

Something that the concept of a rating curve ignores is the effect of unsteadiness, or variation with time. In a flood event the slope of the water surface will be different from that for a constant stage, depending on whether the discharge is increasing or decreasing. Figure 1 shows the increased surface slope as a flood approaches the gauging station. As the flood increases, the surface slope in the river is greater than the slope for steady flow at the same stage, and hence, according to conventional simple hydraulic theory explained above, more water is flowing down the river than the rating curve would suggest. The effect of this is shown on Figure 2, with the discharge marked Q_{rising} obtained from the horizontal line drawn for a particular value of stage. When the water level is falling the slope and hence the discharge Q_{falling} is less.

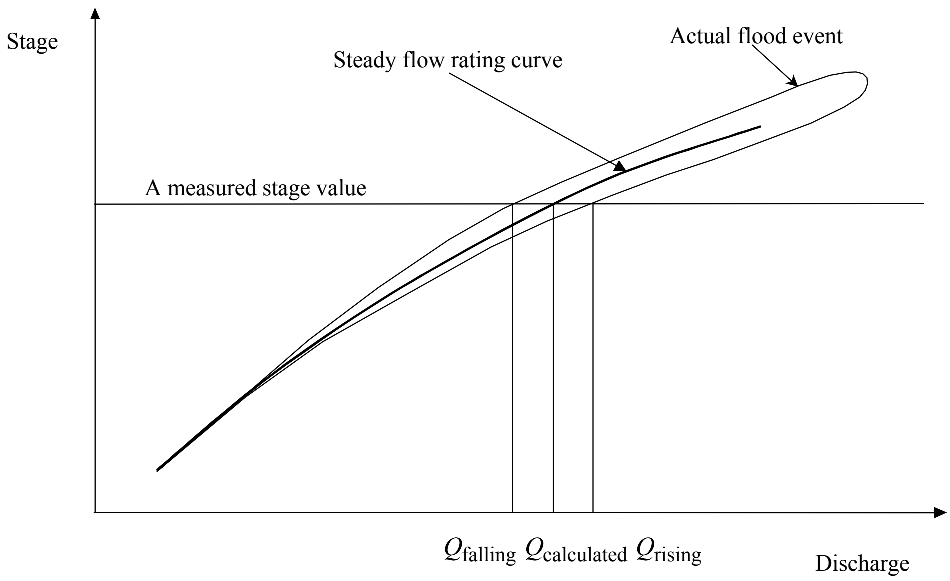

Figure 2. Stage-discharge Diagram showing the Steady-flow Rating Curve and a Possible Trajectory of a Particular Flood Event.

The effects of this might be important – the peak discharge could be significantly underestimated during highly dynamic floods, and also since the maximum discharge and maximum stage do not coincide, the arrival time of the peak discharge could be in error and may influence flood warning predictions. Similarly water-quality constituent loads could be underestimated if the dynamic characteristics of the flood are ignored, while the use of a discharge hydrograph derived inaccurately by using a single-valued rating relationship may distort estimates for resistance coefficients during calibration of an unsteady flow model.

Fenton (1999) developed a method which allows for the effects of unsteadiness on the flow inferred from stage records and rating curves, which was part of the work described in more detail in Fenton and Keller (2001). The long wave equations were considered, and terms cross-eliminated, giving a differential equation for the discharge Q as a function of time t and the instantaneous slope of the free surface $\partial\eta/\partial x$, where η is the free surface elevation. As this is usually not known, a good hydraulic approximation can be made by assuming that the flood wave is following an advection-diffusion equation, so that the space derivative can be expressed in terms of time derivatives of the stage, which can be obtained from the stage record. The expression and the procedure were somewhat complicated.

A considerable simplification can be had by incorporating the low-inertia approximations which lead to the advection-diffusion model. The result is the explicit approximation

$$Q = Q_{\mathrm{r}}(\eta)\sqrt{\underbrace{\underbrace{1}_{\text{Rating curve}} + \frac{1}{c\bar{S}}\frac{d\eta}{dt}}_{\text{Jones formula}} \underbrace{- \frac{D}{c^3\bar{S}}\frac{d^2\eta}{dt^2}}_{\text{Diffusion term}}}, \tag{7}$$

where Q is the actual discharge at the gauging station, $\overline{S}$ is the bed slope, $Q_r(\eta)$ is the discharge for the station as a function of stage obtained from the rating curve, c is the kinematic wave speed, given by

$$c = \frac{1}{B}\frac{dQ_r}{d\eta}, \tag{8}$$

where B is the width of the surface of the stream and $dQ_r / d\eta$ is the gradient of the rating curve, and where D is the coefficient of diffusion from advection-diffusion theory

$$D = \frac{Q_r}{2B\overline{S}}, \tag{9}$$

This is an extension to Jones' method for correcting for the effects of unsteadiness (see Henderson, 1966, p393), which assumed that the flood wave moved as a kinematic wave without diminution. The extra diffusion term here has been obtained by allowing for diminution in by using an advection-diffusion level of approximation.

5 A THEORETICAL EXAMPLE

A numerical solution has been obtained for the particular case of a fast-rising and falling flood in a stream of 10km length, of slope 0.001, which has a trapezoidal section 10m wide at the bottom with side slopes of 1:2, with a Manning's friction coefficient of 0.04. Above a downstream weir the depth of flow is 2m, while carrying a flow of $10\,\mathrm{m}^3/\mathrm{s}$. The incoming flow upstream was linearly increased to $100\,\mathrm{m}^3/\mathrm{s}$ over 60 mins and then reduced to the original flow over another 60 mins. To provide a basis for comparison a simulation was performed using the full long wave equations. Initially the backwater curve problem was solved with a fourth-order Runge-Kutta scheme. Then the long wave equations in the channel were solved numerically using an accurate method based on the method of characteristics. The spatial approximation was by cubic splines, with errors proportional to the fourth power of the step size. In time a simple first-order Euler method was used, but with Richardson extrapolation to the limit, so that the truncation errors were proportional to the third power of the step size. Both space and time steps were reduced until the computations had converged to at least the visual accuracy of the results presented here. At a station halfway along the waterway the computed stages were recorded, which are the data one would normally have, as well as the computed discharges so that the accuracy of this work could be tested. The accuracy of the program has been established in several consulting projects. It can, for example, simulate the steady state of the Murray River from Hume Dam to Mildura for 30 days with no point being in error by more than a millimetre.

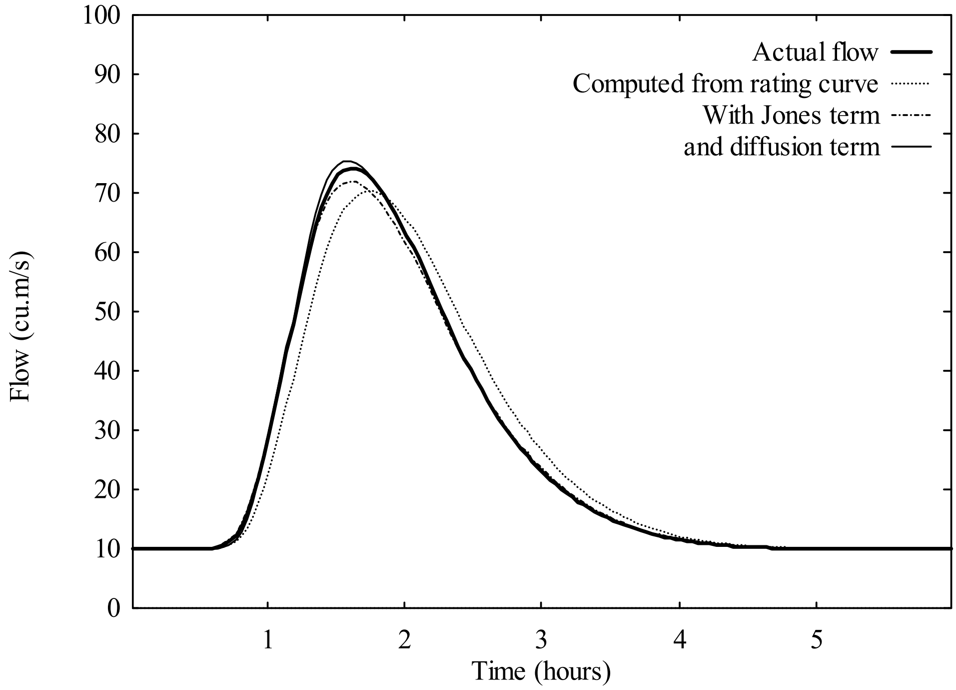

Figure 3. Simulated Flood with Hydrographs Computed from Stage Using Three Levels of Approximation in (7). The "Actual Flow" is the Hydrograph Computed from the Simulation of the Long Wave Equations.

With the results obtained from the simulation the differential equation corresponding to the full long wave equations was solved. Then the three levels of approximation in (7) were applied. The highest approximation, including the diffusion term, agreed very closely indeed with the results from the differential equation, better even than expected. Results are shown on Figure 3, where the differential equation results have not been shown separately.

It can be seen that the application of the diffusion level of approximation has succeeded well in obtaining the actual peak discharge. The results are not exact however, as our derivation depended on the diffusion being sufficiently small that the interchange between space and time differentiation would be accurate. In the case of a stream such as the example here, diffusion is relatively large, and our results are not exact, but they are better than the Jones method at predicting the peak flow.

Nevertheless, the results from the Jones method, shown in Figure 3, are interesting. A widely held opinion is that the Jones formula is not accurate. Indeed, here it is clear that in predicting the peak flow it is not. However, over almost the entire flood it is accurate, and it predicts the time of the flood peak well, which is also an important result. It shows that both before and after the peak the "discharge wave" leads the "stage wave", which is of course in phase with the curve showing the flow computed from the stage graph and the rating curve. As there may be applications where it is enough to know the arrival time of the flood peak, this is a useful property of the Jones formula. Near the crest, however, the rate of rise becomes small and so does the Jones correction. Now, and only now, the inclusion of the extra diffusion term in (7) gives a significant correction to the maximum flow computed, and is quite accurate in its prediction here that the real flow is some 5% greater than that which would have been calculated just from the rating curve. In this fast-rising example the application of the unsteady corrections seems to have worked well and to be justified. It is not much more difficult to apply the diffusion correction than the Jones correction, both being given by derivatives of the stage record.

6 ESTIMATING WHEN TO APPLY THE UNSTEADY CORRECTIONS

The necessity of correcting for unsteadiness is considered by considering the relative contribution of the advective (Jones) and diffusive terms in (7). As the terms will be relatively small compared with the first term, 1, the square root term is expanded using the binomial theorem to first order:

$$Q \approx Q_{\mathrm{r}}(\eta)\left(1+\frac{1}{2}\frac{1}{c\overline{S}}\frac{d\eta}{dt}-\frac{1}{2}\frac{D}{c^3\overline{S}}\frac{d^2\eta}{dt^2}\right), \tag{10}$$

where a factor of $1/2$ has appeared in front of each term. This approximation is accurate to within 1% if the corrections are less than 25%.

Equation (10) is written as $Q \approx Q_{\mathrm{r}}(\eta)\left(1+\Delta_A+\Delta_D\right)$, thereby introducing the terms *advection correction* Δ_a and *diffusion correction* Δ_d respectively, defined by

$$\Delta_a = \frac{1}{2}\frac{1}{c\overline{S}}\frac{d\eta}{dt} \quad \text{and} \quad \Delta_d = -\frac{1}{2}\frac{D}{c^3\overline{S}}\frac{d^2\eta}{dt^2}. \tag{11}$$

In a typical flood these corrections will contribute at different times, the advection correction contributing when the rate of change is highest, when the second derivative will usually be small, and the diffusion correction will be greatest in the vicinity of the flood peak, when the other contribution is small.

The advection correction will be positive when the stage is increasing and negative when decreasing, such that relative to the stage graph both rising and falling limbs of the hydrograph, and accordingly the peak, are always earlier than the hydrograph taken from the rating curve. The diffusion correction will be positive near a flood peak, as $d^2\eta/dt^2$ is negative there, and so the general result can be stated that the flow maximum of every flood occurs earlier and is larger than the flow computed from a rating curve.

In many situations this will not be very significant, however, and formulae to estimate the magnitude of these corrections are now obtained. The kinematic wave speed c is approximated by the wide-channel result from Manning's equation $c \approx 5/3 \times U$ where U is the mean water velocity, then the advection correction is given by

$$\Delta_a \approx \frac{3}{10}\frac{d\eta/dt}{\bar{S}U}, \tag{12}$$

while for the diffusion correction, also substituting $D = Q/2B\bar{S} = UA/2B\bar{S}$, and introducing the mean depth $h = A/B$:

$$\Delta_d \approx -\frac{27}{500}\frac{h}{U^2\bar{S}^2}\frac{d^2\eta}{dt^2}. \tag{13}$$

These equations are in terms of the mean velocity, and might be useful in practice to evaluate the importance of these corrections, as velocity can be computed from the discharge and the cross-sectional area, both of which are usually known at a gauging station. Alternatively, an order of magnitude estimate for U could be had by assuming a value typical of river or flood flows in the region, say, 2m/s. The velocity is probably the quantity that varies least over all conditions.

From both equations (12) and (13) the corrections are largest for rivers where the conditions change quickly but are otherwise slow-moving with a mild slope. In fact, these conditions are often mutually exclusive, such that slow-moving rivers are likely to be slow to rise and fall. Nevertheless, it is quite possible that there are stations where the corrections are necessary. More insight into what determines the magnitude of the corrections can be gained if the flow velocity is related to river characteristics. It is assumed that the velocity is given by Manning's equation for a wide channel of mean depth h: $U = 1/n \times h^{2/3}\bar{S}^{1/2}$, which in practice would require knowledge of the roughness n. Substituting into (12) gives

$$\Delta_a \approx \frac{3}{10}\frac{n}{\bar{S}^{1.5}h^{2/3}}\frac{d\eta}{dt}. \tag{14}$$

Using another wide-channel approximation $d\eta/dt \approx dh/dt$ (the exact expression is actually $dh/dt = (1 - h/B \times dB/d\eta) \times d\eta/dt$), in a whimsical result the effects of the geometry of the channel can be expressed simply as the cube root of the depth:

$$\Delta_a \approx \frac{9}{10}\frac{n}{\bar{S}^{1.5}}\frac{d}{dt}\left(h^{1/3}\right). \tag{15}$$

This reveals how the effect is greatest for rough (n large), gently sloping ($\bar{S}$ small) channels where the depth is large and changes quickly, as expected.

Following a similar procedure for the diffusion correction, using the wide-channel approximation:

$$\Delta_d \approx -\frac{27}{500}\frac{n^2}{\bar{S}^3}\frac{1}{h^{1/3}}\frac{d^2\eta}{dt^2}.$$

In fact, if it is attempted to isolate the variation with depth, the whimsy seems to continue, for *at a flood peak*, where $dh/dt = 0$:

$$\Delta_d^{\text{peak}} \approx -\frac{81}{1000}\frac{n^2}{\bar{S}^3}\frac{d^2}{dt^2}\left(h^{2/3}\right)\Bigg|_{\text{peak}}, \tag{16}$$

where, except for the 1000 in the denominator, everything on the right of this equation, including the differential operator, is the square of a corresponding term in the formula for the advective correction, (15). There is no obvious physical reason for this, however the result is interesting for it shows that the diffusive correction depends on the same quantities, on n, on $\bar{S}$, and on mean depth h in a roughly similar manner to the advective correction. This is not obvious from the original definitions in (11).

This suggests introducing the quantity with units of time

$$T = \frac{nh^{1/3}}{\bar{S}}, \tag{17}$$

such that the effects of unsteadiness could be estimated by monitoring the magnitudes of

$$\Delta_a \approx \frac{9}{10}\frac{dT}{dt} \quad \text{and} \quad \Delta_d^{\text{peak}} \approx -\frac{81}{1000}\left.\frac{d^2T^2}{dt^2}\right|_{\text{peak}}, \tag{18}$$

where the numerical coefficients could be rounded to 1 and 0.1 respectively, remaining in keeping with the approximate nature of the calculation.

In most applications of this work, the diffusive correction at the peak of a flood, in this form or from (16), might be the only calculation performed. It is a value that could be computed for typical floods at each gauging station, to test whether or not the unsteadiness correction should subsequently be applied to stage records there. If the value were less than a certain desired value, say 0.02 or 0.05 (2% or 5% of the maximum), then it might be decided that no further effort need be made to correct for unsteadiness. On the other hand, if the peak correction were large enough, then it might be decided to apply the unsteady corrections in the form of (7) or (10) to every stage record from that gauging station.

7 CONCLUSIONS AND RECOMMENDATIONS

For intermediate and high flows the slope of the surface, as well as stage, is a determinant of flow. The concept of a unique rating curve is flawed in principle, but in practice it is often accurate enough. Ideally it would be best always to use two gauges and to measure the slope, which would automatically correct for backwater effects from downstream and unsteady effects at the time of flood propagation.

In the absence of such measurements a method has been developed which gives a correction to the flow calculated from a rating curve, which is effectively an extension of the well-known Jones method for allowing for effects of the variation of stage with time. Formulae have been given to estimate when unsteady effects are worth correcting for, and an example presented. For many rivers the unsteady effects are small, and so the use of two gauges, although ideal, is often not justified, and results from conventional rating curves are quite acceptable.

8 ACKNOWLEDGEMENTS

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