One-dimensional flow modelling and a case study of the River Rhine

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ABSTRACT: The simulation of rivers by one-dimensional models is examined. The German Federal Waterways Engineering and Research Institute's model CasCade was applied to a comprehensive case study for a long non-uniform section of the Rhine river with extensive floodplains. Results obtained show that the one-dimensional long-wave equations are more accurate than often believed. Large real-time data sets allow model calibration and optimisation beyond the matching of field data, and reproducing unsteady wave characteristics over the whole flow regime. It is then shown theoretically that, possibly surprisingly, river junctions require no special treatment in one-dimensional modelling. However some improvements are suggested – for curved rivers, over flood-plains, and in the treatment of friction. It is concluded that one-dimensional models, including the results of this paper, preserve a balance between requirements of accuracy and simplicity, and have much to offer in the understanding of river systems.

1 INTRODUCTION

1.1 *Rhine River engineering – the CasCade model*

Figure 1. Rhine River at Ketsch near Heidelberg, showing large flood plains, branches and lakes, being flow effective during high flows, but ineffective during low flows (Source: Landesmedienzentrum Baden-Württemberg)

The Rhine river ($1320\,\mathrm{km}$, $2000\,\mathrm{m^3s^{-1}}$ average discharge) is one of the longest and most important rivers in Europe (www.iksr.de). Its watershed is highly populated (about 50 million people) and connects the world's largest sea harbour (Rotterdam) with the inland European markets and its large and important industrial complexes. Ships up to $3000\,\mathrm{t}$ can navigate one of the most frequented waterways of the world from Rotterdam to Basel ($800\,\mathrm{km}$). The Rhine is also an important ecological habitat, source of major drinking water supplies and is of recreational and cultural importance. River engineering therefore has to guarantee optimised navigation conditions and flood protection combined and balanced with natural preservation (Figure 1). The German Federal Waterways Engineering and Research Institute (BAW, www.baw.de) is the central technical and scientific governmental agency of the German Federal Ministry of Transport, Building and Urban Development and the Federal Waterways and Shipping Administration. Its responsibilities are the operation and maintenance of federal waterways such as defining dimensions of fairways, regulating waterways structures or measures for stabilising the river bed. The assessment concerning the short-term and long-term hydraulic-morphological effects as well as the water management and environmental aspects is realised using physical as well as one-dimensional and multi-dimensional hydro-numerical models complementing one another.

A computer model for a $500\,\text{km}$ long section of the river Rhine has been developed and operated by the BAW using the in-house software CasCade, a one-dimensional, hydro-numeric modelling system for unsteady, unconfined flow conditions in branched or meshed networks of rivers, canals and flood fields. CasCade covers a major part of the river Rhine and therefore allows a unique system-wide river analysis. Its applications regard the following objectives:

- *Real time flow dynamics - online computation of water level elevation and depth*: CasCade models can easily be connected to the Rhine water gauge online-database, which is continuously updated with real-time data. Unsteady model runs include the previous flow history and use the actual water level time-series as boundary conditions at open boundaries. Additional water gauge data furthermore allows for a real-time model accuracy check. Waterway authorities already operate these models to improve fairway definitions, thus improving navigation conditions.

- *Navigation dynamics - computation of ship travel times*: CasCade post-processors allow for detailed data analysis. For example mean flow velocities can be computed for the navigation channel and used for further analysis of the navigation dynamics, thus transformed into ship travel times. Knowledge of unsteady hydrodynamic features therefore allows for improved logistical handling and better traffic control.

- *River bed dynamics - computation of mean shear stress in main channel*: CasCade post-processors may also compute section-wise shear stress. A system-wide analysis covering the whole flow regime allows for defining critical sections regarding erosion or sedimentation, especially after large floods, thus providing important data reducing the amount of costly field measurements and improving maintenance programs.

- *River engineering dynamics - simulation of impacts of regulating waterways structures*: The continuously updated and operated model allows for more accurate impact prediction beyond the calibrated flow regimes (Figure 2). The model is setup with all geometrical features regarding the whole flow regime. Although changes of regulating waterways structures are included only in an aggregated sense and not as higher-dimensional features, reasonable predictions are possible due to the additional unsteady calibration of the model.

- *Modelling dynamics - nesting/coupling optimisation*: A good knowledge of the system wide river flow behaviour, covering local characteristics over large discharge regimes allows optimising the complementary or coupled usage of additional models (i.e. more dimensional models). More detailed but costly 3-D hydrodynamic or morphology models are therefore applied especially to these sections and flow-regimes, where the 1-D model is passing its limits of applicability, exceeding desired accuracies.

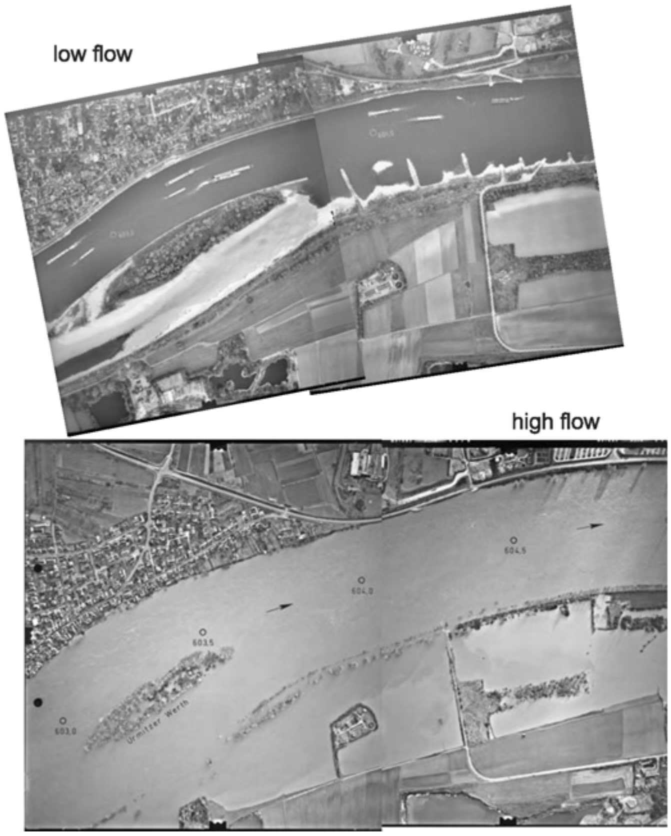

Figure 2. Short section of Rhine River showing waterway structures (groynes) and natural characteristics (islands), resulting in a strong change of the already complex flow characteristics during increasing flows

1.2 *Case-study – section from Speyer to Worms*

A brief illustration of CasCade capabilities and limitations is shown for the application on the Rhine section from Speyer to Worms (Rhine-km 400.6 - 443.4). Along this streamwise 42.8 km there is an approximate bed height difference of 6 m resulting in an average bed slope of 0.015%. The width of the main channel ranges from 200 to 350 m and the depth from 2 m (during low flow conditions at $400 - 900\,\text{m}^3\text{s}^{-1}$) to 5 m (during medium flow conditions at $900 - 1600\,\text{m}^3\text{s}^{-1}$). This river section is characterised by its strong heterogeneity with partly very large flood plains (a few kilometres wide), including side branches, lakes and other retention areas (Figure 1). Another part is almost canal-like, where the river passes the cities of Mannheim and Ludwigshafen including the junction with a major tributary river, the Neckar (Figure 3).

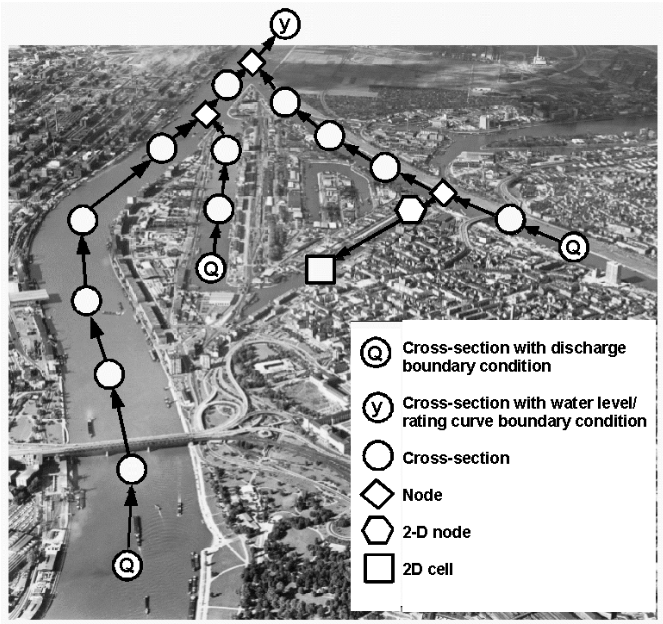

Figure 3. Rhine-Neckar junction and model schematisation as branched network

The BAW maintains an extensive database, where topography and bathymetry data are available from recent laser and sonar scans respectively, with high spatial resolution. Cross-sections are extracted at each $100\,\mathrm{m}$ distance in the streamwise direction and used for the 1-D model setup. River sections separated by islands or old river arms are modelled as separate branches, leading to a looped network model (Figure 3). Aerial photographs, geographical and geological maps, as well as navigational charts, allow locating and including important hydraulic structures and features in the model. The model Pre-Processor allows for subdividing cross-sections according to main roughness characteristics: the main channel, the floodplains, groyne fields or flow ineffective areas (encroachment approach). Model calibration is usually done using longitudinal water level measurements from recent field campaigns where ships travelled with the flow velocity and recorded position and elevation with high accuracy for low and medium flow conditions. Further unsteady calibration is done using the continuous water gauge recordings, with stored data of the last 10 years. Results show, that standard deviation of measured and calculated water level elevations in the main channel are around $2-4\,\mathrm{cm}$ for low flow conditions, $4-10\,\mathrm{cm}$ for medium flow conditions compared with the field campaign data and even less difference for the unsteady data compared with the water gauge time-series, also for high flow events. Nevertheless higher discrepancies can be observed at sections with strong curvature, at junctions and large floodplains not necessarily only due to limitations of 1-D modelling, but also to restrictions in the actual governing equations as seen by running sensitivity analyses.

Improvements to the 1-D numerical models are proposed, focusing on calibrating the model to adequately represent the observed flow characteristics and not only matching field data. Besides the attempt to improve the 1-D model itself the presented modifications should also clearly be seen as extensions beyond the 1-D approach as an interface to either more dimensional models or coupling to other models (hydrology, water quality, sediment transport, *etc*.).

1.3 *Research needs and achievements*

The simulation of flows and waves in rivers for a long time has been undergoing a development in terms of the sophistication of models which are actually applied, which has been related to computational facilities available. Once only steady gradually-varied problems were studied. Then unsteady one-dimensional problems were able to be simulated, followed by two-dimensional problems. Methods such as the development of the $k-\varepsilon$ model and then Large Eddy Simulation have meant that three-dimensional turbulent problems can now be simulated over limited domains. In general however, for practical problems the data requirements are inordinate, and the actual boundary conditions, the manner in which the fluid encounters resistance, are little better modelled than in the one-dimensional case.

Often field data is limited, and it is superfluous to try to model in a more sophisticated manner than the data warrants. This leads to the desirability of using a relatively simple model, but by performing inverse modelling (calibration) to extract the gross features of physical parameters. The use of one-dimensional modelling enables rather more physical understanding. With more sophisticated models the plethora of results often mean that physical intuition and understanding, perhaps the main benefit of computation, are not forthcoming. Also, in many rivers such as the Rhine, the physical extent is such that only a one-dimensional model is feasible.

In the case of the one-dimensional formulation, however, there remain areas where theoretical developments have not developed as they might. This paper examines several aspects of the one-dimensional modelling used by CasCade and by other one-dimensional computational models and tries to show that there is still much that one-dimensional modelling has to offer, especially if some of the suggested improvements are made:

- The one-dimensional long wave equations are examined, and it is shown that they have fewer essential approximations than has generally been believed.
- Generalised long wave equations are presented for curved rivers. The effects of curvature are examined.
- The problem of a junction of two rivers is considered, and it is shown that no special treatment is necessary; the only modification of the model necessary is that dynamic momentum flux of incoming streams should be included, and that this will be small in many cases.
- Friction in rivers is considered and it is observed that one-dimensional modelling is commensurate with the information usually to hand. However there has long been room for improvement in the modelling of friction. It is shown that the Weisbach friction formulation has advantages over the traditional Gauckler-Manning-Strickler version, and it is shown how compound friction can easily and rationally be included in 1-D models.

2 THE LONG WAVE EQUATIONS

2.1 *Equations for straight channels*

The one-dimensional equations usually used to model flows and waves in rivers are the long wave or Saint-Venant equations. They are a pair of partial differential equations for mass and momentum conservation, shown here in terms of the surface elevation η and the discharge Q, depending on distance along the channel x and time t, where the stream is assumed to be straight. The equations are

$$\frac{\partial \eta}{\partial t}+\frac{1}{B}\frac{\partial Q}{\partial x}=\frac{q}{B} \tag{1a}$$

$$\frac{\partial Q}{\partial t}+2\beta\frac{Q}{A}\frac{\partial Q}{\partial x}+\left(gA-\beta\frac{Q^2B}{A^2}\right)\frac{\partial \eta}{\partial x}= \beta\frac{Q^2B}{A^2}\bar{S}-gA\frac{Q\,|Q|}{K^2}+\beta_q\,q\,u_q-\frac{Q^2}{A}\frac{\partial \beta}{\partial x}, \tag{1b}$$

where

q	is inflow volume per unit length,
B	is width of surface,
A	is cross-sectional area,
β	is the Boussinesq momentum coefficient,
$\bar{S}$	is mean bed slope at a section,
K	is conveyance, giving the effect of friction,
β_q	is the Boussinesq coefficient of inflow, and
u_q	is the inflow x-component of velocity.

Fenton (2006) suggested that the derivation of the equations can be made with few essential assumptions. The only non-trivial ones are not particularly limiting. They are:

- Variation in the streamwise direction is slow, such that pressure at a point is given by the equivalent hydrostatic pressure due to the water above that point, which is almost everywhere the case, except in the vicinity of local structures;
- The friction forces are given by an empirical formulae such as the Weisbach or Gauckler-Manning-Strickler formulae for the section as a whole, which in the absence of very detailed knowledge of the roughness, nature, and drag force of the bed, is a reasonable one; and
- The integral of the square of the horizontal velocity over a section is approximated by $\beta Q^2/A$, where β is a coefficient of magnitude about 1.1; this is a rough approximation, but the factor β only occurs in terms which are of a relative magnitude of the square of the Froude number $\mathbf{F}^2 = Q^2B/gA^3$, which is small for many streams.

Generally the one-dimensional long wave equations are a good compromise between requirements of accuracy, and the scarcity of data, especially the nature of shear forces. Equation (1b) contains features worthy of note:

- The quantity $\bar{S}$ actually contains all the details of the underwater topography, and with it the equations are valid for non-prismatic sections (Fenton (2006). It is

  $$\bar{S} = -\frac{1}{B}\int_B \frac{\partial Z}{\partial x}\,dy + \text{term from vertical diverging walls}, \tag{2}$$

  where Z is the bed elevation, such that $\bar{S}$ is obviously the mean bed slope at a section. Some formulations include a term like $\partial A/\partial h$, usually known as the non-prismatic contribution, but it seems simpler to represent it in terms of the mean slope. The form of equation (2) shows that to evaluate it accurately, one needs to have a detailed knowledge of the underwater topography. In the case of the Rhine River, such a detailed knowledge is available. But often this is not the case, however, and the term is written only as a representative slope of the river as a whole. As this term is largely counterbalanced by the friction term, which is also usually poorly known, this is quite justifiable.

- The friction term $gAQ\left|Q\right|/K^2$ is shown in terms of the conveyance K, which is obtained from friction laws:

$$\begin{aligned} K &= \frac{1}{n}\frac{A^{5/3}}{P^{2/3}} = k_{\mathrm{St}}\frac{A^{5/3}}{P^{2/3}} \\ &= \sqrt{\frac{8g}{\lambda}}\frac{A^{3/2}}{P^{1/2}} = C\frac{A^{3/2}}{P^{1/2}} \end{aligned} \tag{3}$$

  in which P is the wetted perimeter, and n, k_{St}, λ, and C are respectively the roughness coefficients of Manning, Strickler, Weisbach, and Chézy. The friction term is based on boundary stress, which is clear from the derivation of the Chézy and Weisbach forms – what is most notable about this formulation is that it is *not* based on energy, and the dimensionless term $Q\left|Q\right|/K^2$ is *not* an "energy slope", as described by many sources.

- The term $\beta_q\, q\, u_q$ is the contribution to momentum flux from lateral inflow. In many cases it is poorly known, when it is not zero, but in the case of a tributary, the β_q is technically necessary to express correctly the momentum transport. Some texts write the whole term as $q\left(u_q - Q/A\right)$, which is not correct (and is based on a double counting of the momentum of the inflow as it enters and leaves the control volume).

2.2 *Channels curved in plan*

Fenton and Nalder (1995) derived the one-dimensional equations for the case where the river is curved. A long-stream co-ordinate s was used, generally expected to pass down the perceived centre of the stream. The equations were obtained in terms of the discharge Q, as for the straight-channel case, but in terms of the cross-sectional area A rather than the surface elevation η, which in this case can vary across the channel due to centripetal acceleration. The difference between the equations obtained and the straight channel equations are linear terms in two quantities, $\kappa n_{\mathrm{m}} = n_{\mathrm{m}}/r$ and $\kappa\bar{n} = \bar{n}/r$, where $\kappa = 1/r$ is curvature of the longitudinal s axis along the river and r is the radius of curvature, n_{m} is the transverse distance between the s co-ordinate axis chosen and the centre of the free surface, and $\bar{n}$ is the distance between the s axis and the centroid of the cross-section. In addition to the approximations for the straight-channel case, also terms proportional to $\left(n_{\mathrm{m}}/r\right)^2$ and $\left(\bar{n}/r\right)^2$ are ignored. That is, the extra approximation is that the radius of curvature be rather larger than the channel width. This level of approximation is similar to the usual one for the conventional formulation of the Saint-Venant equations for channels presumed to be straight, where the approximation is that (depth / disturbance length)$^2 \ll 1$. The equations are

$$\left(1 - \kappa n_{\mathrm{m}}\right)\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial s} = q, \tag{4a}$$

$$\begin{aligned} &\left(1 - \kappa\bar{n}\right)\frac{\partial Q}{\partial t} + \beta\frac{Q}{A}\left(2 + 3\left(\kappa n_{\mathrm{m}} - \kappa\bar{n}\right)\right)\frac{\partial Q}{\partial s} + \\ &\left(\frac{gA}{B} - \beta\frac{Q^2}{A^2}\left(1 + 2\left(\kappa n_{\mathrm{m}} - \kappa\bar{n}\right)\right)\right)\frac{\partial A}{\partial s} \\ &= gA\bar{S} - gA\frac{Q\left|Q\right|}{K^2}\left(1 - \kappa\bar{n}\right) + \beta_q\, q\, u_q \end{aligned} \tag{4b}$$

where terms involving $d\kappa/ds$ have been ignored, and the inflow term has been simplified. The latter equation is slightly different from that in Fenton and Nalder (1995). It includes the momentum coefficient β and the friction term has been written more correctly in terms of conveyance rather than "energy slope". The coefficient β may be important particularly in a curve where the longitudinal velocity is greater on the inside of a curve than on the outside so that β is rather larger than 1. In the original paper no attempt was made to solve the field equations to obtain the longitudinal or lateral velocity distributions – the momentum formulation does not require that knowledge in detail.

Equations (4) are of a very similar nature to those for straight channels, equations (1). The only differences are the presence of the dimensionless coefficients κn_{m} and $\kappa\bar{n}$ in terms of curvature and cross-channel dimensions. The behaviour of waves and flows, the development of numerical methods, the behaviour and properties of those methods, and the rewriting of software, all should be able to be relatively simply done.

3 RIVER JUNCTIONS

3.1 *The canonical problem – rectangular channels*

Almost all research on river junctions has been for the case of steady flow in two rectangular channels with co-planar bottoms, where one enters the other, whose width remains unchanged thereafter. The belief that the problem is difficult, even for this relatively simple geometry, has stultified extension to more complicated geometries and to the general unsteady case. Here we examine a momentum approach to the problem, to guide our extension to more general problems. The energy principle does not contribute much – all that it yields is the energy loss, which is not as important as the relation between upstream and downstream depths that is obtained from momentum considerations.

Consider a rectangular stream entering another at an angle δ as shown in Figure 4. The main stream has

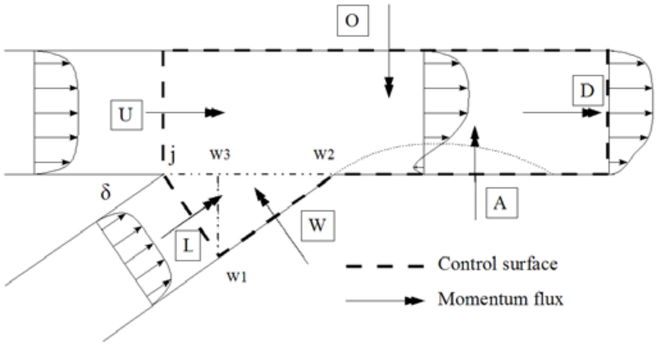

Figure 4. Definition diagram for momentum analysis of a junction of two uniform rectangular streams

the same width upstream and downstream of the junction. The plan of a vertical-sided control volume is shown by a heavy dashed line and the flow force contributions are shown by double-headed arrows. The downstream face of the control surface has not been taken across the region of separation, but it is supposed sufficiently far downstream that the velocity defect due to the separation has been substantially dissipated.

The flow force **M**, consisting of a static component, the hydrostatic force and a dynamic component, the momentum flux, across a planar part of a control surface, where the velocity crosses the section perpendicularly is $\mathbf{M} = \rho\left(gAh_{\mathrm{c}} + \beta Q^2/A\right)\hat{\mathbf{n}}$, where symbols are the same as above, with the addition of h_{c}, the depth of the centroid of the section below the surface, such that Ah_{c} is the first moment of area of the section about a transverse axis at the water level, and $\hat{\mathbf{n}}$ is a unit vector, normal to the section, directed outwards from the control volume.

The vertical boundaries across which momentum flows, with the corresponding reference letters on Figure 4, are a transverse **U**pstream face across the stream, a similar **D**ownstream face, a **L**ateral stream entering at an angle of δ to the main stream, the side of the main stream **O**pposite to the inflowing stream, and the side **A**djacent to it. The analysis we are about to perform assumes that the square of the Froude number of the incoming flow at L is not large so that the disturbances to the free surface are also not large.

It is the *dynamic* contribution from L that is the most important factor in the analysis. If we consider a momentum balance *transverse* to the main stream, then by a simple theorem from hydrostatics, the component of a static force on any surface is equal to the component of the static force on a projection of that surface perpendicular to the component direction (*e.g.* §2.6 of White (2003). This means that the static component from L in the transverse direction is approximately equal to that on the rectangular section from the junction point j to w_3, and the component from the wall W is approximately equal to that on the projection w_2w_3. Hence the static component of the whole lateral entry region is approximately that of a hypothetical wall from j to w_2, just as if the tributary were not there. This leaves the only transverse contribution that of the dynamic contribution from the tributary, which can only be balanced by the force contribution from O being slightly larger than that from A and jw_2, so that the water level along O opposite the tributary will be slightly higher and the water level along the junction and A will be slightly lower – the transverse dynamic contribution causes the river locally to tilt slightly transversely. While this seems to violate the initial assumption that the water level does not vary much because of the tributary, the overall deduction that the effect is small for small Froude number is true.

Similarly, if we consider longitudinal flow force, along the main stream, it can be seen that the longitudinal static contribution on L will be approximately equal to the contribution due to its projection, the vertical rectangle marked on the surface by w_1w_3, while the contribution on the wall W is only static, and has the same projection area but its contribution is the reverse of that due to L, and so the static contributions almost completely cancel each other out, and the only flow force contribution to the inflowing stream is its dynamic component in the longitudinal direction, which will be usually relatively small.

To quantify this, we write the longitudinal momentum balance, assuming that on each contributing face the water level is constant, but not for the moment assuming that $h_{\mathrm{W}} = h_{\mathrm{L}}$, to give, after re-arrangement:

$$\tfrac{1}{2}Bh_{\mathrm{D}}^2 + \frac{\beta_{\mathrm{D}}Q_{\mathrm{D}}^2}{gBh_{\mathrm{D}}} - \left(\tfrac{1}{2}Bh_{\mathrm{U}}^2 + \frac{\beta_{\mathrm{U}}Q_{\mathrm{U}}^2}{gBh_{\mathrm{U}}}\right) - \cos\delta\,\frac{\beta_{\mathrm{L}}Q_{\mathrm{L}}^2}{gbh_{\mathrm{L}}} + \tfrac{1}{2}b\cos\delta\left(h_{\mathrm{W}}^2 - h_{\mathrm{L}}^2\right) = 0, \qquad (5)$$

in which b is the width of the tributary. A good approximation is that the water level in both confluent streams is the same, such that $h_{\mathrm{L}} = h_{\mathrm{U}}$, which is supported by evidence (Hager (1987, citing Favre and Vischer). The last term in (5) shows mathematically the approximate balance between the contribution of the wall and that due to the lateral stream. However the actual depth along the wall is not known *a priori*, although in many cases, the water surface drops very little as the tributary enters the main stream, and so $h_{\mathrm{W}} \approx h_{\mathrm{L}}$. As a first approximation a factor $\gamma = h_{\mathrm{W}}/h_{\mathrm{L}}$ could be introduced in the spirit of Hager (1989), and a numerical value slightly less than 1 assigned. The result is an equation connecting h_{U} and h_{D} in terms of all the β and Q of the three component streams. If we were to make the further approximation that all the dynamic terms, of magnitude proportional to the square of the Froude number, then the solution of the momentum equation is simply $h_{\mathrm{D}} \approx h_{\mathrm{U}}$.

3.2 *Unsteady flow through junctions of arbitrary section*

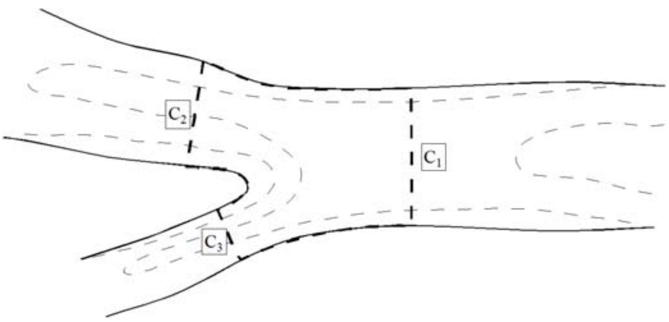

Figure 5. Definition diagram for momentum analysis of a junction of two irregular natural streams

Consider a natural river junction as shown in Figure 5. The treatment here is of some generality, so we abandon the nomenclature of Upstream and Downstream, and prefer to identify the channels as 1, 2 and 3. Shown is a possible control volume with vertical planar flow boundaries C_1, C_2, and C_3.

In this case the solid boundaries that give contributions to the momentum from hydrostatic forces, are irregular, complicated, and very difficult to evaluate, if indeed the bottom topography is known at all well. However for rectangular channels in both transverse and longitudinal directions it was seen above that significant cancellations and simplifications occurred. That suggests further exploration for the case of irregular geometry. The reason is to be found in Gauss' Divergence Theorem in one of its lesser-known forms, (#2.61, equation 3 of Milne-Thomson (1968) to replace the integral of the pressure contributions over *all* parts of the control volume by an integral of the pressure gradient through the volume, as used by Fenton (2006) in a derivation of equations (1). Gauss' relation is

$$\int_{\mathrm{CS}} p\,\hat{\mathbf{n}}\,dS = \int_{\mathrm{CV}} \nabla p\,dV. \tag{6}$$

The basic approximation throughout river hydraulics is that the pressure distribution is equal to the equivalent static head of water above, such that at any point with elevation z, and where η is the water surface elevation above, $p = \rho g\,(\eta - z)$, and ∇p is the vector with components $\rho g\,(\partial\eta/\partial x, \partial\eta/\partial y, -1)$. As we will not consider vertical momentum here, we can take just the x and y components of the right side of (6), such that $\nabla_2 = (\partial/\partial x, \partial/\partial y)$ and use these results to give the pressure force $\mathbf{P}_2$ in the two-dimensional horizontal plane on the fluid in the control volume:

$$\mathbf{P}_2 = \int_{\mathrm{CS}} \nabla_2\,p\,dV = \rho g \int_{A_{\mathrm{s}}} h\,\nabla_2\eta\,dA_{\mathrm{s}} \tag{7}$$

where the last term is the integral over the surface area A_{s} of the junction, of the depth h times the surface gradient. In this form the contribution can be approximated readily without gross error, in the spirit of much river hydraulics. If we take the component of the net pressure force along part of the control volume, and just consider the x-component along the volume, in a slice of length Δx and elementary volume ΔV, the term becomes, as $\partial h/\partial x$ is closely constant across the width,

$$\rho g \int_{A_{\mathrm{s}}} h\,\frac{\partial\eta}{\partial x}\,dA_{\mathrm{s}} = \rho g \frac{\partial\eta}{\partial x}\Delta V = \rho g \frac{\partial\eta}{\partial x} A\Delta x, \tag{8}$$

and this results in the term $gA\,\partial\eta/\partial x$ in the momentum equation for the channel, (1b). This underlying simplicity leads us to suggest that for all the physical processes governing motion in the junction, there is nothing which requires special modelling if momentum is considered. Energy is quite different, because the mixing of two streams at an angle to each other causes extra losses. For momentum, however, all processes are similar to those which occur in the body of the river itself. The process of friction drag is similar, although extra currents will cause some minor differences.

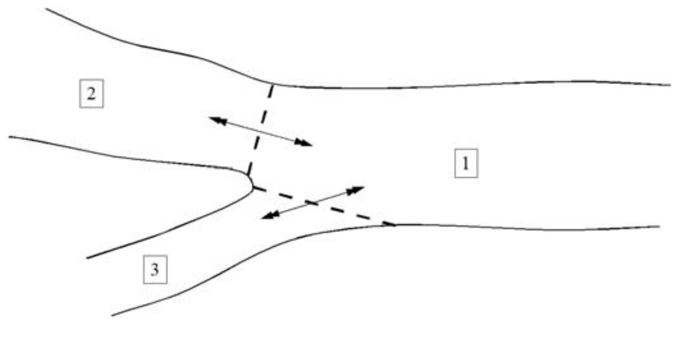

Figure 6. The schematisation which can be used in practice at a junction of two irregular natural streams

In the case of a junction of two rectangular channels, the water surface slope does vary around the junction, and for higher speed flow, it does dip as it passes the sharp corner w_2 in Figure 4. However, for natural junctions as in Figure 5 we assert that the river surface shows little unusual variation, and that it becomes possible to do away with the concept of a special control volume at the junction. It seems easier and in keeping with the usual one-dimensional approach to rivers if instead, the river junction is considered as a junction between different one-dimensional streams, as shown in Figure 6. This might have been the simplest and most obvious approach from the beginning, but the junction problem has been traditionally believed to be too complicated for such a simple representation.

The CasCade model already incorporates this representation in its looped network formulations. A major problem for the actual case study, as presented previously, has been the lack of data regarding the lateral inflow volume flux. There is no rating curve existing for the Neckar tributary nearby the junction, because of the influence by the Rhine river superimposed on the unknown discharge control by upstream hydropower plants. Furthermore it is not possible to develop a rating curve for a long section upstream of the junction, due to the influence of the Rhine discharge on Neckar water levels and even more important, the influence of the Neckar discharge on Rhine water levels. A calibrated CasCade model was therefore used to artificially develop a rating surface at the junction point. Given downstream water level and discharge, together with given upstream discharge allows computing the Neckar flow and upstream water levels for both. An *a priori* calculation of the rating surface using Equation (5) would improve the modeling of junctions considerably, which still has to be proven.

In view of this apparent simplicity, it seems that the only consideration at a junction is to include the dynamic momentum contribution from the tributaries. Equation (5) for rectangular channels clearly contains a dynamic contribution from the inflow that becomes, after multiplying through by g:

$$\cos\delta\,\frac{\beta_{\mathrm{L}} Q_{\mathrm{L}}^2}{b h_{\mathrm{L}}} \tag{9}$$

that does affect the main stream. Here we examine the adequacy of the inflow term $\beta_q\, q\, u_q$ in the general momentum equation (1b) to represent this. We multiply by the length of the finite computational length Δx in the downstream river. It is clear that $\beta_q = \beta_{\mathrm{L}}$, $q\Delta x = Q_{\mathrm{L}}$, and $u_q = \cos\delta\, Q_{\mathrm{L}}/b h_{\mathrm{L}}$, and the expressions are equivalent. Hence we suggest that the only effort or modification necessary to allow for a river junction is to add the dynamic momentum contribution as an inflow contribution, such that over the first computational length the term $\beta_q\, q\, u_q$ be incorporated as a total contribution of $\cos\delta\, \beta_{\mathrm{L}} Q_{\mathrm{L}}^2/A_{\mathrm{L}}$ where the subscripts L pertain to the lateral stream and δ is the angle it makes with the downstream channel.

4 THE CALCULATION OF FRICTION

4.1 *Friction formulations*

The Gauckler-Manning-Strickler equation is empirical – it has no theoretical justification. It is simple and describes friction for a river cross-section as a whole. Rouse (1937) wrote

> "… hydraulicians have been forced to rely to a great extent upon empirical relationships for the expression of energy loss due to turbulence; and as empiricism is at best a rather haphazard means to an end, such relationships in themselves are seldom likely to suggest the correct nature of underlying physical principles. Thus it is that the subject of hydraulics now finds itself somewhat hobbled in its thought by those very methods which contributed greatly to its prestige in past generations.

However, an alternative was already at hand, Keulegan (1938) noted that the theoretical investigations of Prandtl and von Kármán, and the experimental work of Nikuradse, led to rational formulas for velocity distribution and hydraulic resistance for turbulent flow in circular pipes in terms of the dimensionless Weisbach friction coefficient λ. He went on to show how similar rational formulas could be deduced for open channels, although rather than allowing for a general treatment by allowing for the local depth to vary over a section, he adopted the overall section hydraulic radius as the characteristic length.

The optimism of the 1930s was misplaced – the Gauckler-Manning-Strickler law continues to appear as some sort of universal standard, such that experimental and field results are related to it, arbitrary formulae are used to apply it to compound sections and more general problems, and books and tables are published suggesting that friction can be calculated from visual similarity with other rivers. There are a number of arbitrary formulae for calculating the effective friction coefficient for a section, not based on rational criteria. Indeed the ASCE Task Force on Friction Factors in Open Channels (1963) wrote:

> "Many engineers have become accustomed to using Manning's n for evaluating frictional effects in open channels. At the present stage of knowledge, if applied with judgement, both n and λ are probably equally effective in the solution of practical problems. The design engineer who prefers to use n in his (*sic*) computations should continue to do so, but he should recognize the limitations on his method ... It is believed that experimental measurements of friction in open channels over a wide range of conditions are better correlated and understood by the use of λ. Furthermore, λ is commonly used by engineers in many other branches of engineering and probably provides the only basis for pooling all experience on frictional resistance in both open and closed conduits. It is recommended,

therefore, that engineering teachers and research workers emphasize the use of the friction factor λ ...

Clearly the Task Force believed that Manning's n was sufficiently entrenched in practice that it could continue to be used but that it was desirable for researchers and teachers to move the profession in the direction of λ.

4.2 *A drag-force approach to channel friction*

The resistance to flow in open channels has often been thought of, incorrectly, in energy terms, giving rise to the concept of energy gradient. Energy dissipation processes are internal to the flow and are complicated and difficult to describe and quantify. However the concepts of the Chezy resistance coefficient C and the closely allied Weisbach coefficient λ are derived from considerations of wall stress, hence momentum. Also, research into the effects of vegetation on flow has used a momentum approach, in terms of drag forces on individual elements, which has usually then been converted to an equivalent overall Gauckler-Manning-Strickler roughness by processes whose validity is doubtful.

Using momentum/drag it is possible to unify effects due to boundary roughness, bed-forms, vegetation, and structures – using the empirical Gauckler-Manning-Strickler law that is not possible. Consider the combined drag force ΔT on a number of discrete roughness elements and/or vegetation elements in an elemental slice of channel of length Δx:

$$\Delta T = \frac{\rho}{2} \sum C_\mathrm{D} v^2 a, \tag{10}$$

where the summation is over all the drag-producing elements, each of projected area a normal to the flow of local velocity v and with drag coefficient C_D. Theoretically the sum contains a huge number of terms of small quantities (every sand grain, for example). It is not intended that the sum ever be actually evaluated, it is here only as a representation. If the local velocity v is assumed given by a factor γ related to the mean horizontal velocity in the channel Q/A, then $v^2 = \gamma Q^2/A^2$, and

$$\Delta T = \frac{\rho}{2} \frac{Q^2}{A^2} \sum C_\mathrm{D} \gamma a. \tag{11}$$

The *mean* shear stress on the channel wall is given by (p63, Montes (1998):

$$\tau_0 = \rho \frac{\lambda}{4} \frac{(Q/A)^2}{2}, \tag{12}$$

where λ is the Weisbach friction coefficient, so that, using $\tau_0 = \Delta T / P \Delta x$ gives λ expressed in terms of the drag components:

$$\lambda = \frac{4 \sum C_\mathrm{D} \gamma a}{P \Delta x}. \tag{13}$$

This gives a fluid-dynamical explanation for the behaviour of λ, and shows how it is actually made up of *local* components, and how a rational theory of channel friction becomes possible:

- In general, the elements of the sum $\sum C_\mathrm{D} \gamma a$ will vary around the boundary, as the bed material changes, as vegetable matter of a different kind grows at the edges, as the local value of γ depends on the local depth, and so on.
- Clearly the drag coefficient terms can be added linearly so that we could write it as a *finite* sum of various contributions

$$\lambda = \lambda_1 + \lambda_2 + \ldots, \tag{14}$$

  where 1, 2, *etc* might denote gravel on bed, reeds at the side, and so on, each of which can be determined separately by research.
- In the case of the Gauckler-Manning-Strickler equation it is not at all clear how individual components should be combined, and there are several alternatives.
- With some research and experience it would be possible to calculate the sum, and hence the friction coefficient λ, from a knowledge of bed materials of the same form.
- In general, the sum and hence the friction factor λ, will be a function of depth and flow. It, unlike Manning's n (Strickler's k_St), is not expected to be constant. Its variability is an expression that we are modelling the actual physics. We do not expect a magic constancy for such a complicated problem.

In the case of a channel with particles of sand or gravel, we can use previous research which has obtained expressions for λ. Colebrook and White proposed an implicit equation for λ in a pipe in terms of relative roughness k/D, the equivalent sand grain roughness divided by the pipe diameter D,and the Reynolds number $\mathbf{R} = UD/\nu$. For channels this can be used with $D = 4A/P$ (Henderson (1966, p93). As λ appears implicitly in the equation, it has been thought of as being more difficult than it really is. To try to overcome this Moody (1944) produced a well-known chart for λ as a function of $\mathbf{R}$, with k/D as a parameter; it is not always known that most of the results are simply those corresponding to the Colebrook and White equation. The best solution of all is to use the explicit approximation produced by Haaland (1983):

$$\lambda = \frac{1}{1.8^2} \log_{10}^{-2} \left(\left(\frac{k/D}{3.7} \right)^{10/9} + \frac{6.9}{\mathbf{R}} \right), \tag{15}$$

where again, $D = 4A/P$ for a channel.

4.3 *Use in practice*

In equation (3) the conveyance K in terms of λ is given by

$$K = \sqrt{\frac{8g}{\lambda}} \frac{A^{3/2}}{P^{1/2}} \tag{16}$$

so that in the 1-D momentum equations (1b) and (4b) the friction term appears as

$$-\frac{gAQ\,|Q|}{K^2} = -\lambda P \frac{Q\,|Q|}{8A^2}, \tag{17}$$

with an extra curvature factor in the latter case. The friction coefficient λ appears linearly, as does perimeter P, as they are directly related to the force on the boundary, to which the whole term corresponds. If the boundary were a compound boundary, for example, or with different roughness elements, one could write the friction term in terms of a sum

$$-\left(\sum_i \lambda_i P_i\right) \frac{Q\,|Q|}{8A^2}. \tag{18}$$

If this approach were adopted, it may be less necessary to resort to 2-D or 3-D modelling, as the characteristics of the whole cross-section are incorporated in this 1-D model.

5 CONCLUSIONS

The simulation of rivers has been examined, by one-dimensional models in general, and by the program CasCade in particular. Extensive data management allows for analysis of real-time flow dynamics coupled with navigation dynamics, river bed dynamics and impact analysis of regulating waterway structures. The governing equations have been shown to require few limiting assumptions. Furthermore the problem of the confluence of two or more natural streams has been considered, and the problem has been shown to be able to be approximated by conventional one-dimensional approaches, with the incorporation of the momentum from and to any tributaries or effluent streams. The problem of simulating floodplain flows was considered, and it was concluded that even though the one-dimensional approach does not solve the problem as well as might be desired, irregularities of roughness, channel and geometry are such that no great sophistication is warranted and further developments of a simple nature are warranted. Methods for calculating friction have been considered, and it has been suggested that the Weisbach formulation is characterised by its universal applicability beyond the main channel hydraulics. It is concluded that the knowledge of frictional characteristics in the bed and the floodplains of a river, and the detailed geometry of both, are usually so limited that the one-dimensional model represents a good compromise between requirements of fidelity of simulation and the greatly increased effort and intrinsic limitations of highly accurate simulation by two- and three-dimensional models.

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