Institute of Hydraulic and Water Resources Engineering, Vienna University of Technology

River Engineering

Tutorial Sheet 6 – Discharge Measurement and Rating Curves

1. In lectures we had a scheme for obtaining the mean velocity in the vertical by measuring the velocity at two points. A simpler scheme is where the velocity is measured at a single point $0.6\,h$ from the surface. Show that the velocity at this point ($0.4\,h$ above the bottom) is a close approximation to the mean velocity for velocity profiles in (a) & (b) by integrating to find the mean velocity, and then finding the value of z for which the velocity is equal to the mean.

   a. The Prandtl-von Kármán law for turbulent flow over a rough bed:

   $$u = \frac{u_*}{\kappa} \log \frac{z}{z_0}.$$

   You will need the result that $\int \log z \, dz = z \log z - z + C$. (Ans: $z/h = e^{-1} = 0.37$, such that the relative depth of measurement should be 0.63).

   b. The simple $1/7$ law, sometimes used as a simpler model for turbulent velocity distributions:

   $$u = U_{\max} \left(\frac{z}{h} \right)^{1/7},$$

   where $U_{\max}$ is the surface velocity. (Ans: $z/h = 0.39$, relative depth 0.61)

   c. And, do it for the general power law

   $$u = U_{\max} \left(\frac{z}{h} \right)^{\nu}.$$

   (Ans: $z/h = (1+\nu)^{-1/\nu}$. Plot it and be astonished how little it varies for $0 < \nu < 0.25$. Then take the limit as $\nu \to 0$ and be astonished that it approaches the value $e^{-1} = 0.37$, the same as for the logarithmic law. This is a glorious coming-together of mathematics, for it is Euler's formula for e: $\lim_{x\to 0}(1+x)^{1/x}$!)

2. The Australian water industry uses a non-*SI* unit for flow, namely Megalitre per day (Ml/d) and hydrographers sometimes use a unit of velocity of km/day for calibrating their propeller meters and presenting their data. This is not as silly as it sounds.

   a. Verify that a cube $10\,\mathrm{m} \times 10\,\mathrm{m} \times 10\,\mathrm{m}$ contains 1 Megalitre.

   b. It is often said that 1 Megalitre is roughly the size of a $50\,\mathrm{m}$ Olympic swimming pool. Make some estimates of other dimensions and test the truth of that statement.

3. Hydrographers sometimes use a unit of velocity of km/day for calibrating their propeller meters and presenting their data. This is not as silly as it sounds.

   a. Verify that if velocities in km/day are integrated over cross sectional areas specified in m^2, the result is directly Ml/d. The velocity in km/day gives a practical idea of the distance that the water will travel in a day.

   b. Verify that the velocity in km/day is also roughly the velocity in $\mathrm{cm\,s^{-1}}$, also useful for practical considerations, and show that a velocity of 30 km/day is $34.7\,\mathrm{cm\,s^{-1}}$

4. Write a short essay, possibly in bullet point form and possibly without mathematics, on Rating Curves, describing what they are, what axes might be used to represent and approximate them, and what problems do they have in not doing what they are supposed to do, giving the actual instantaneous flow.

5. Describe, again in bullet point form, and using a figure of surface height (stage) plotted against discharge, to describe the behaviour of the stage-discharge relationship for a typical flood event in an alluvial river, where bed "particles" are free to move.