Institute of Hydraulic and Water Resources Engineering, Vienna University of Technology

River Engineering

Tutorial Sheet 5 – Gradually-varied flow equation

The differential equation governing the water depth in a steady flow in a stream is

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{\tilde{S} - Q^2/K^2}{1 - \beta \mathsf{F}^2}, \qquad \text{(Gradually-varied flow equation)}$$

where h is the water depth, $\tilde{S}$ is the bottom slope, Q is the discharge, $K = 1/n \times A^{5/3}/P^{2/3}$ is the conveyance, and $\beta\mathsf{F}^2$ (F the Froude number) expresses the fluid inertia effects, which can be ignored for a channel on a sufficiently mild slope.

1. A canal has a bed slope of 1×10^{-4}, $n = 1/k_{\mathrm{St}} = 0.02$, and flows uniformly with a depth of $2\,\mathrm{m}$. It is proposed to install a barrage on it which will raise the water level there to $h = 2.5\,\mathrm{m}$ so as to enable irrigation of the surrounding land. You are to investigate how far upstream the enhanced water levels will affect the channel.
   a. Using a spreadsheet or any other software, solve the GVFE numerically using Euler's method, starting with $h_0 = 2.5\,\mathrm{m}$ and stepping upstream backwards in x (*i.e.* Δx is negative) with steps of $2000\,\mathrm{m}$ (until the water level is nearly within $1\,\mathrm{cm}$ of the normal depth).
   b. How far upstream is it? Are you surprised?
   c. Plot the actual surface elevation, which is h plus the local bed elevation.
   d. Repeat your calculations with steps of $1000\,\mathrm{m}$. How do your two answers compare – at $x = -20\,\mathrm{km}$ for example?
   e. Refine your solution at every $2000\,\mathrm{m}$, using Richardson extrapolation, which in this case (for Euler's method) can be written for a point x:
   $$h_{\text{Improved}}(x) = 2h_{\Delta/2}(x) - h_{\Delta}(x).$$
   f. Use Samuels' formula
   $$h = h_0 + He^{\gamma x}, \quad \text{where} \quad \gamma \approx \frac{10}{3}\frac{S_0}{h_0}$$
   and plot the results.
2. Experiment with a different slope, say a steep slope of $S = 1 \times 10^{-3}$, with a normal depth of $h_0 = 1\ \mathrm{m}$ (note: Q is different) and for a checked-up depth of $1.5\ \mathrm{m}$ at the barrage to see how the length of the backwater changes. You will have to use smaller steps of $100\ \mathrm{m}$.