Institute of Hydraulic and Water Resources Engineering, Vienna University of Technology

River Engineering

Tutorial Sheet 2 – Propagation of flood waves

1. **Wave speed of very long waves**

   Can you convince yourself, in your brain and in your stomach, that if we have a nonlinear equation governing the passage of volume of water,

   $$\frac{\partial V}{\partial t} + U_{\mathrm{r}}(A)\frac{\partial V}{\partial x} = 0,$$

   where the advection velocity is $U_{\mathrm{r}}(A) = Q_{\mathrm{r}}(A)/A$ and $Q_{\mathrm{r}}(A)$ is the steady uniform discharge, that small disturbances v will propagate such that

   $$\frac{\partial v}{\partial t} + c(A)\frac{\partial v}{\partial x} = 0,$$

   at an advection velocity of $c = \mathrm{d}Q_{\mathrm{r}}/\mathrm{d}A > U_{\mathrm{r}}$?

2. To be specific: the speed at which the main body of disturbances move in rivers is the *very long wave speed* c. The Kleitz-Seddon formula for c is

   $$c = \frac{dQ_{\mathrm{r}}}{dA} = \frac{1}{B}\frac{dQ_{\mathrm{r}}}{d\eta},$$

   where Q_{r} is a function of area A or water surface elevation η respectively, given by any of the expressions

   $$Q_{\mathrm{r}} = \begin{cases} Q_{\mathrm{r}} & \text{measured;} \\ A\,k_{\mathrm{St}}\left(\frac{A}{P}\right)^{2/3}\sqrt{S}, & \text{Gauckler-Manning-Strickler ;} \\ A\sqrt{\frac{g}{\Lambda}\frac{A}{P}S}, & \text{Chézy-Weisbach .} \end{cases}$$

   a. Show that the value from the Gauckler-Manning-Strickler expression can be written in terms of the mean velocity of flow, $U = Q/A$ as

   $$c = U\left(\frac{5}{3} - \frac{2}{3}\frac{A}{P}\frac{dP}{dA}\right).$$

   b. It is more convenient to express the derivative in terms of the local water height. Using the fact that $dA/dh = B$ (interpret that physically) show that

   $$c = \frac{5}{3}\times U \times \left(1 - \frac{2}{5}\frac{A}{PB}\frac{dP}{dh}\right).$$

   The quantity dP/dh is easily shown to be $2\sqrt{1+m^2}$, where m is the batter slope, so that this might have a value of, for $m = 2$ say, of $2\sqrt{5} \approx 4.5$. This means that the relative contribution due to the perimeter changing is roughly $2 \times A/BP$, which is the twice the mean depth divided by the wetted perimeter, which will be small for typical wide shallow channels.

3. Consider a trapezoidal canal of bed width $10\,\mathrm{m}$ and batter slopes $2:1$ (H:V) excavated to a slope of 1 in $10\,000$, and $n = 1/k_{\mathrm{St}} = 0.03$. For a depth of $2.5\,\mathrm{m}$,

   a. Calculate the discharge Q and the mean velocity in the channel (*Ans:* $Q = 18.3\,\mathrm{m^3\,s^{-1}}$, $U = 0.49\,\mathrm{m\,s^{-1}}$).

   b. Calculate the very long wave speed and also calculate it using the wide-channel approximation. Calculate the "dynamic wave speed", $C = \sqrt{gA/B}$. (*Ans:* $c = 0.69\,\mathrm{m\,s^{-1}}$, $0.81\,\mathrm{m\,s^{-1}}$, $C = 4.3\,\mathrm{m\,s^{-1}}$).

   c. Calculate the estimated time of travel of a disturbance over a distance of $10\,\mathrm{km}$. (*Ans:* $4\,\mathrm{h}$).

4. In January 2011 the lecturer stayed with his mother for a month on the riverine plains of inland Australia. There were huge rains and a major flood (in the middle of the Australian summer). Mother and son had to evacuate from a farm $25\ \mathrm{km}$ away into the village as she was a possible medical emergency case and the only bridge was about to be blocked by floodwater. Two days later at 5:30 in our emergency accommodation, sirens rang, and we were compulsorily evacuated on the only road remaining open, as the levee banks surrounding the village were in danger of bursting. All very professionally interesting!

(a) General view of flood approaching lecturer's home village

(b) Water level has reached the bridge in the village. If only someone knew how to calculate the effect ...

(c) GPS measuring water level and anxiously numerically differentiating with respect to time and extrapolating. The most important calculation!

(d) Village surrounded, levees in danger of bursting, people (inc lecturer and mother) about to be compulsorily evacuated at 5:30 tomorrow

(e) Eucalyptus tree exerting resistance

(f) The electricity supply for hundreds of thousands

The flood wave shown in Figure (a) was several kilometres wide and many kilometres long. The peak took about 5 days to travel $120\ \mathrm{km}$. Estimate the mean water velocity. The land falls by about $40\ \mathrm{m}$ in $75\ \mathrm{km}$. Estimate k_{St} for the floodplain if the mean depth were estimated to be $1\ \mathrm{m}$.

Assuming that the flood took one day to rise initially, calculate the dimensionless period gST/U_0 and compare with the limit $gST/U_0 \gtrsim 10$ to see that our neglect of time derivative and fluid inertia terms were justified. Now calculate

$$\text{Relative importance of diffusion} = \frac{1}{2c_0 S}\frac{\eta_{\rm P}-\eta_0}{t_{\rm P}-t_0}$$

to see if our neglect of diffusion was justified. Use $\eta_{\rm P}-\eta_0 \approx 4$ m.

Solution

$$\begin{aligned} S &= \frac{40}{75\times 1000} \approx 5\times 10^{-4} \\ c &= \frac{120\times 1000}{5\times 24\times 3600} = 0.28 \text{ ms}^{-1} \\ U &= \frac{3}{5}\times 0.28 \approx 0.17 \text{ ms}^{-1} \\ 0.17 &= k_{\rm St}\times 1^{5/3}\sqrt{0.0005} \\ k_{\rm St} &\approx 7.5 \end{aligned}$$

Yes, this is very low, but the water is flowing over grassland, fields, bushes, trees, *etc*., with many obstructions such as irrigation canals.

Dimensionless period – neglect of terms in the equations justified?

One day to rise, period = 2 d,

$$gST/U = 10\times 5\times 10^{-4}\times 2\times 24\times 3600/0.168 \approx 5100,$$

a very long wave indeed. Our neglect of time derivative and fluid inertia terms very well justified.

Relative importance of diffusion

$$\begin{aligned} \text{Relative importance of diffusion} &= \frac{1}{2cS}\frac{\eta_{\rm P}-\eta_0}{t_{\rm P}-t_0} \\ &= \frac{1}{2\times 0.28\times 5\times 10^{-4}}\times\frac{4}{1\times 24\times 3600} \\ &\approx 0.17 \end{aligned}$$

This “ball park” estimate (thank you, USA) shows some importance of diffusion. At the other end of the river, where my home village is, the flood took a week or two to pass, and had been quite diffused – spread out in space and time. The velocity of propagation we calculated is probably still reasonable.

5. The river itself runs beside the village. The photograph (e) is of a eucalyptus tree (“River Red Gum”) about $90\,$cm in diameter, near the main stream, where, of course, the local fluid velocity is expected to be rather larger. Estimate the height of the stagnation mound on the upstream face – the lecturer thinks it is about $5\,$cm. Use Bernoulli’s law to estimate the velocity of the water at the surface.

$$U = \sqrt{2g\Delta z} = \sqrt{2\times 10\times 5/100} = 1.0 \text{ ms}^{-1},$$

which agrees with the lecturer’s visual estimate at the time.

Any calculation of the force on the tree would probably not be important ...

In any case, the tree is a good example of the iconic tree of the huge river network of inland south-eastern Australia, `https://en.wikipedia.org/wiki/Eucalyptus_camaldulensis`.