Institute of Hydraulic and Water Resources Engineering, Vienna University of Technology

River Engineering

Tutorial Sheet 1 – Resistance laws for steady uniform flow

1. Gauckler-Manning-Strickler
   a. Plot a graph of Strickler's relationship $k_{\mathrm{St}} = 6.7\sqrt{g}/D^{1/6}$ for D from $0.1\,\mathrm{mm}$ to $30\,\mathrm{cm}$ (logarithmic D axis, linear k_{St} axis). Keep for future use?

   b. Compare the two values you get for $D = 1\,\mathrm{cm}$ and $2\,\mathrm{cm}$. Comment.

   c. Do you have an explanation for the remarkable insensitivity of k_{St} to changes in boundary grain size? (If you do, please tell the lecturer. We will discuss this.).

2. For a trapezoidal cross-section, where the bottom width is W, the depth is h, and the side slopes are (H:V) $m : 1$,
   a. Show the following properties:

   $$\begin{aligned} &\text{Top width } B = W + 2mh \\ &\text{Area } A = h\,(W + mh) \\ &\text{Wetted perimeter } P = W + 2\sqrt{1+m^2}h \end{aligned}$$

   b. Verify that the general relationship $B = dA/dh$ holds true for this type of section.

3. Use the Gauckler-Manning-Strickler formulation and the results for a trapezoidal section to obtain an expression for the steady uniform discharge as a function of depth h:

$$Q = k_{\mathrm{St}} \frac{\left(h\,(W+mh)\right)^{5/3}}{\left(W + 2h\sqrt{1+m^2}\right)^{2/3}} \sqrt{S_0} \qquad (1)$$

4. A commonly-encountered problem in river and canal hydraulics is to compute the normal (uniform flow) depth for a given discharge by numerically solving the transcendental equation you obtained in question 3. Here we consider methods for solving the problem:
   a. A traditional method is to plot the relationship and read the solution off. However, this is not suited to computer programs.

   b. Trial and error – substitute numerical values of h in until the required discharge is obtained, also not suited to computer programs.

   c. Solve the equation by a numerical method for solving transcendental equations such as direct iteration, bisection, or Newton's method, or most simply, using Solver in Excel.

   d. The direct iteration scheme in the lectures works well:

$$\begin{aligned} h_0 &= \left(\frac{Q}{k_{\mathrm{St}} B_0 \sqrt{S}}\right)^{3/5} \\ h_{n+1} &= \left(\frac{Q}{k_{\mathrm{St}}\sqrt{S}}\right)^{3/5} \times \frac{P^{2/5}(h_n)}{A(h_n)/h_n} = \left(\frac{Q}{k_{\mathrm{St}}\sqrt{S_0}}\right)^{3/5} \frac{\left(W + 2h_n\sqrt{1+m^2}\right)^{2/5}}{W + mh_n}, \end{aligned}$$

   which is then evaluated repeatedly.

   e. Obtain the corresponding iteration scheme for the Chézy-Weisbach formulation.

$$h_{n+1} = \left(\frac{\lambda Q^2}{8gS_0}\right)^{1/3} \frac{P^{1/3}(h_n)}{A(h_n)/h_n} = \left(\frac{\lambda Q^2}{8gS_0}\right)^{1/3} \frac{\left(W + 2h_n\sqrt{1+m^2}\right)^{1/3}}{W + mh_n}.$$

f. Check: is that dimensionally correct? *Always* check for dimensional correctness.

g. Note that such direct iteration does not work for all equations – it did work here because we made sure that our equation was a weakly varying function of h.

5. Consider a canal with $W = 10$ m, side slopes of 2:1, a longitudinal slope of 0.0005, $k_{\mathrm{St}} = 50$. For a flow of $20\ \mathrm{m^3 s^{-1}}$, calculate the normal depth h.

   a. If you have access to a spreadsheet with Solver installed, use that to solve the problem.

   b. Use the direct iteration scheme in question 4.d

      (*Ans:* $h = 1.348\,\mathrm{m}$ – if we start with $B_0 = W = 10$, we then obtain $h_0 = 1.418$, $h_1 = 1.344$, $h_2 = 1.349$, $h_3 = 1.348$ m, and it has converged to within a millimetre).