# 10. Sediment transport

Most rivers and canals have beds of soil, of a more-or-less erodible nature, which is composed of particles. Water flowing in such streams has the ability to remove and carry the particles, or to deposit them, hence changing the bed topography. This is of great importance, for example in predicting the scouring around and potential collapse of bridges, weirs, channel banks *etc*., estimating the rate of siltation of reservoirs, predicting the possible form changes of rivers with a threat to aquatic life *etc*..

## 10.1 Sediment stability

### Dimensional analysis

The grains forming the boundary of an alluvial stream can be brought into motion if the fluid forces acting on a sediment particle are greater than the resisting forces. This is usually expressed in terms of mean shear stress on the bed. If the shear stress $\tau$ acting at a point on the flow boundary is greater than a certain critical value $\tau_{\text{cr}}$ then grains will be transported.

The variables which dominate the problem of the removal and transport of particles are:

| | | | | | |
|---|---|---|---|---|---|
| $\rho$ | Density of water | $\mathrm{ML}^{-3}$ | $g$ | Gravitational acceleration | $\mathrm{LT}^{-2}$ |
| $\rho_s$ | Density of solid particles | $\mathrm{ML}^{-3}$ | $h$ | Depth of flow | L |
| $\nu$ | Kinematic viscosity of water | $\mathrm{L}^2\mathrm{T}^{-1}$ | $\tau$ | Shear stress of water on bed | $\mathrm{ML}^{-1}\mathrm{T}^{-2}$ |
| $D$ | Diameter of grain | L | | | |



As we have 7 such quantities and 3 fundamental dimensions involved, there are 4 dimensionless numbers which can characterise the problem. In fact, it is convenient to replace $g$ by $g' = g\,(\rho_s/\rho - 1)$, the apparent submerged gravitational acceleration of the particles, and to replace $\tau$ by the shear velocity $u_* = \sqrt{\tau/\rho}$.

Convenient dimensionless variables, partly found from physical considerations, which occur are

$$\Theta = \frac{u_*^2}{g'D} = \frac{\tau}{(\rho_s - \rho)\,gD},$$
Dimensionless stress or the *Shields parameter*, roughly the ratio of the shear force on a particle to its submerged weight

$$\mathsf{R}_* = \frac{u_* D}{\nu},$$
*Grain Reynolds number*, roughly the ratio of fluid inertia forces to viscous forces on the grain

$$G = \frac{\rho_s}{\rho},$$
*Specific gravity* of the bed material, often $G \approx 2.65$ (quartz)

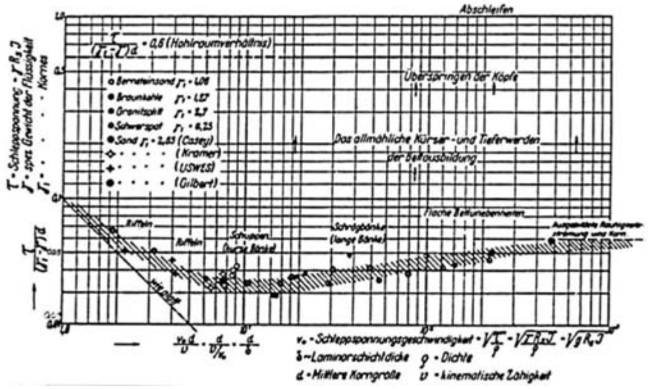
$$\frac{D}{h}$$
Ratio of grain size to water depth



## 10.2 Incipient motion

### The Shields diagram

In the 1930s Albert Shields conducted a number of experiments in Berlin and found that there was a band of demarcation between motion and no motion of bed particles, called incipient motion. He represented this band on a figure of $\Theta$ versus $\mathsf{R}_*$. This is the best-known contribution in the study of sediment stability and movement.



An English translation is given here on p181.



### A better representation in terms of grain size

A problem with this is that the fluid velocity (in the form of shear velocity $u_*$) occurs in both abcissa $\mathsf{R}_*$ and ordinate $\Theta$. It is better to form another dimensionless quantity from which $u_*$ has been eliminated, the dimensionless grain size (see p7 of Yalin & Ferreira da Silva 2001):

$$D_* = \left(\frac{\mathsf{R}_*^2}{\Theta}\right)^{1/3} = D\left(\frac{g'}{\nu^2}\right)^{1/3}.$$

Here we consider what $D_*$ means. If we take the common value of $G = 2.65$, plus $g = 9.8\,\mathrm{ms}^{-2}$, $\nu = 10^{-6}\,\mathrm{m}^2\mathrm{s}^{-1}$, (for 20°C), then we obtain $D_* \approx 2.5 \times 10^4 \times D$ with $D$ in units of metres. For a range of particle sizes we have

| $D_*$ | 0.1 | 1 | 10 | 100 | 1000 |
|---|---|---|---|---|---|
| $D\,(\mathrm{cm})$ | 0.0004 | 0.004 | 0.04 | 0.4 | 4 |

This means that instead of the Shields plot $(\mathsf{R}_*, \Theta)$ we can plot $(D_*, \Theta)$, and as $D_* \propto D$, it is effectively a plot of $(D, \Theta)$, so that to determine whether a particular bed of given particle size $D$ is stable, we can just read $\Theta_{\text{cr}}$ off the figure or calculate it using the formulae that we will present below. There are a number of different results and features of the figure:

- The number of experimental points and an outlined region of experimental points show how variable the results can be. There might be different definitions of what "incipient" motion is – *e.g.* how many particles have to move to be deemed unstable? Shields approach was well-judged, showing a finite band of results.

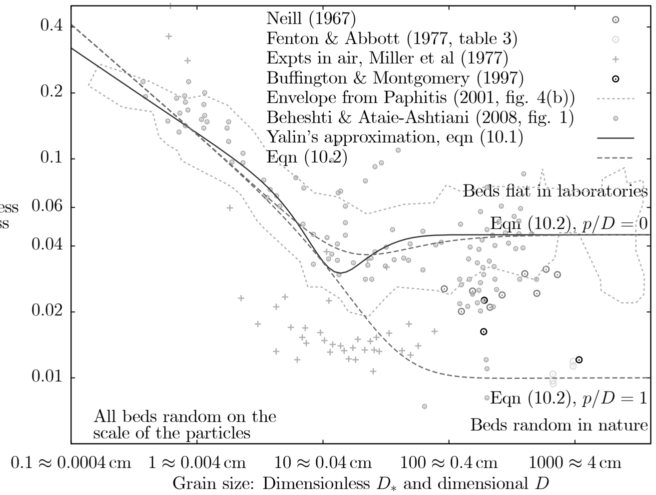
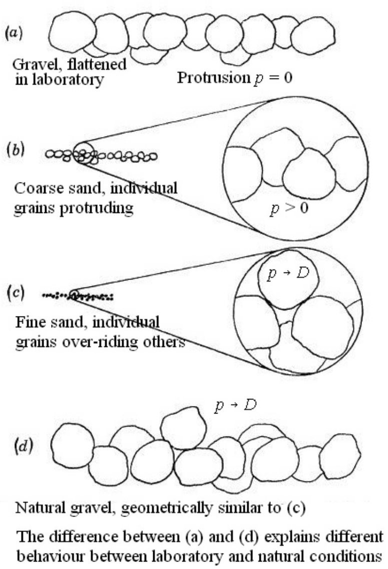



Figure 10.1: Stability diagram, dimensionless stress $\Theta$ plotted against grain size, showing experimental results for $\Theta_{\text{cr}}$ and approximating functions.






- Bagnold (personal communication) suggested that there was probably no fluid mechanical reason for the apparent dip in the data on the Shields diagram itself, rather that there is an artificial geometric effect due to scale, and suggested that the Shields diagram has been widely misinterpreted. For experiments with small particles, while the overall bed may have been flattened, individual small grains may sit on top of others and may project into the flow, so that the assemblage is random on a small scale. For large particles (gravel, boulders, *etc.*) in nature, they too are free to project into the flow, however in the experiments which determined the Shields diagram, the bed was made flat by levelling the tops of the large particles. Hence, there is an artificial scale effect. Grains in nature are free to project into the flow above their immediate neighbours, so that experimental results for large grains with flattened beds are not applicable to natural situations.



- Fenton & Abbott (1977) followed Bagnold's suggestion and examined the effect of protrusion of particles into the stream. Although they did not obtain definitive results, they were able to recommend that for large particles the value of $\Theta_{\text{cr}}$ was more like $0.01$ than $0.045$, which seems to be an important difference, the factor of $1/4$ requiring a fluid velocity for entrainment into the flow of randomly-placed particles to be about half that of the sheltered case.
- Yalin & Ferreira da Silva (2001, p8) present a mathematical approximation to the Shields results for incipient motion, a single curve giving the critical value $\Theta_{\text{cr}}$:

$$\Theta_{\text{cr}} = 0.13\, D_*^{-0.392}\, e^{-0.015\, D_*^2} + 0.045\, \left(1 - \, e^{-0.068\, D_*}\right). \qquad (10.1)$$

Above the curve, for larger values of $\Theta$ (and hence larger velocities or smaller and lighter grains), particles will be entrained into the flow. For large particles the critical shear stress approaches a constant value of $\Theta_{\text{cr}} = 0.045$. The artificial dip in the curve occurs at about $D = 10$, corresponding to a grain size of $0.4\,\text{mm}$, a fine sand.

- A simpler and more general expression can be obtained which includes relative protrusion $p/D$ as a parameter, and which seems to describe all the features of the figure well:

$$\Theta_{\text{cr}} = \frac{0.13 + \gamma(0.035 - 0.025\, p/D) D_*^2}{D_*^{0.5} + \gamma\, D_*^2}, \qquad (10.2)$$

where $\gamma$ is a parameter whose value determines the rate of approach to the large $D_*$ behaviour when $\Theta_{\text{cr}} \to 0.035 - 0.025\, p/D$. The curves in the figure on page 173 were plotted with $\gamma = 0.01$. One might use $p/D = 0$ for levelled structures such as Block Ramps, in natural sediments it is expected that $p/D \approx 1$ would be more applicable.



## Criterion in terms of stream parameters

Now we consider some simple relations to relate this to physical quantities. The shear velocity $u_*$ is a very convenient quantity indeed. If we consider the steady uniform flow in a channel, then the component of gravity force down the channel on a slice of length $\Delta x$ is $\rho g A\, \Delta x\, S$, where $S$ is slope. However the shear force resisting the gravity force is $\tau \times P \times \Delta x$. Equating the two we obtain

$$\tau = \rho g \frac{A}{P} S.$$

In this work it is sensible only to consider the wide channel case, such that $A/P = h$, the depth, giving

$$\tau = \rho g h\, S,$$

or in terms of the shear velocity:

$$u_* = \sqrt{\frac{\tau}{\rho}} = \sqrt{g h\, S},$$

and so in terms of the dimensionless stress:

$$\Theta = \frac{u_*^2}{g' D} = \frac{g h\, S}{g' D} = \frac{S\, h}{(G-1)\, D}. \qquad (10.3)$$

## 10.3 Laboratory experiments and dimensional similitude

Moveable-bed models are much more difficult to design and operate than fixed-bed models, where the major requirement is just that the bed roughness correspond to the full scale.

In experiments with sediment transport, as in other areas of fluid mechanics, it is desirable to have the same dimensionless numbers governing both experimental and full-scale situations. In this case we would like the dimensionless particle size AND the dimensionless shear stress each to have the same values in both model and full scale. Using the subscript m for model and with no subscript for the full scale situation, we then should have

$$D_{*\mathrm{m}} = D_* \quad \text{such that} \quad D_\mathrm{m}\left(\frac{g'_\mathrm{m}}{\nu_\mathrm{m}^2}\right)^{1/3} = D\left(\frac{g'}{\nu^2}\right)^{1/3}.$$

If water is used, as it is most commonly, viscosity $\nu_\mathrm{m} = \nu$ – although note the variation with temperature:

| Temperature °C | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| $\nu_\text{water}$ ( $\mathrm{m^2 s^{-1}} \times 10^6$) | 1.8 | 1.3 | 1.0 | 0.80 |

As gravitational acceleration $g$ is a constant,

$$D_\mathrm{m}\left(G_\mathrm{m}-1\right)^{1/3} = D\left(G-1\right)^{1/3}.$$

If we use the same bed material, $G_\mathrm{m} = G$, the implication is that we should use $D_\mathrm{m} = D$, which might be very difficult.



We also require the same dimensionless shear stress, so from equation (10.3):

$$\frac{S_\mathrm{m}\, h_\mathrm{m}}{\left(G_\mathrm{m}-1\right) D_\mathrm{m}} = \frac{S\, h}{\left(G-1\right) D}.$$

It is difficult to be able to satisfy all the dimensionless numbers, and often loose-bed models can do little other than provide qualitative information (where does deposition occur?) rather than quantitative information (how rapidly does deposition occur?).

## 10.4 Transport of sediment

Consider the concept of a *relative tractive force* at a point, $\tau/\tau_\mathrm{cr}$. If this is slightly greater than 1, only the grains forming the uppermost layer of the flow boundary can be detached and transported. If $\tau/\tau_\mathrm{cr}$ is greater than 1, but less than a certain amount, then grains are transported by deterministic jumps in the neighbourhood of the bed ("saltation"). These modes of grain transport are referred to as *bed-load*. If the ratio $\tau/\tau_\mathrm{cr}$ is large, then grains will be entrained into the flow and will be carried downstream by turbulence. This transport mechanism is known as *suspended-load*. The total transport rate is the sum of the two. The simultaneous motion of the transporting fluid and the transported sediment is a form of *two-phase flow*.

A formula for the criterion of the initiation of suspension is from Van Rijn:

$$\Theta_\text{cr,susp} = \frac{0.3}{1+D_*} + 0.1\left(1-\exp\left(-0.05 D_*\right)\right),$$

and in the limit of large $D_*$, $\Theta_\text{cr,susp} \to 0.1$.



There are a number of transport formulae.

**Meyer-Peter and Mueller (1948):** This is considered to be the first reliable empirical *bed load* transport formula. They performed flume experiments with uniform particles and with particle mixtures. Based on data analysis, a relatively simple formula was obtained, which is frequently used:

$$\frac{q_\text{sb}}{\sqrt{g' D^3}} = 8\left(\Theta - \Theta_\text{cr}\right)^{1.5},$$

where $q_\text{sb}$ is the volume rate of solids per unit width of stream transported as bed load.

**Einstein (1950)** introduced statistical methods to represent the turbulent behaviour of the flow. He gave a detailed but complicated statistical description of the particle motion in which the exchange probability of a particle is related to the hydrodynamic lift force and particle weight.

**Bagnold (1966)** introduced an energy concept and related the sediment transport rate to the work done by the fluid.

$$q_\text{sb} = \beta u_\mathrm{b} D\left(\Theta - \Theta_\text{cr}\right),$$

where $\beta$ is a function of $D_*$, and $u_\mathrm{b}$ is the flow velocity in the vicinity of the bed. In the case of a rough turbulent flow, $\beta \approx 0.5$. Yalin & Ferreira da Silva (2001, p9) state that this formula is preferred, as it is simple, as accurate as any, and reflects the meaning of the bed-load rate.

**Engelund and Hansen (1967)** presented a simple and reliable formula for the total load transport in rivers.



## 10.5 Bedforms

### Two-dimensional

In most practical situations, sediments behave as non-cohesive materials, and the fluid flow can distort the bed into various shapes. The interaction process is complex. At low velocities the bed does not move. With increasing flow velocity the inception of movement occurs. The basic bed forms encountered are *ripples* (usually of heights less than $0.1\,\mathrm{m}$), *dunes*, *flat bed*, *standing waves*, and *antidunes*. At high velocities chutes and step-pools may form. Typical bed forms are summarised in Figure 10.2 below.

Bed forms for steady flow over a sand bed can be classified into

- Lower transport regime with flat bed, ribbons and ridges, ripples, dunes and bars,
- Transitional regime with washed-out dunes and sand waves,
- Upper transport regime with flat mobile bed and sand waves (anti-dunes).

When the bed form crest is perpendicular to the main flow direction, the bed forms are called transverse bed forms, such as ripples, dunes and anti-dunes. Ripples have a length scale much smaller than the water depth, whereas dunes have a length scale much larger than the water depth. Ripples and dunes travel downstream by erosion at the upstream face (stoss-side) and deposition at the downstream face (lee-side). Antidunes travel upstream by lee-side erosion and stoss-side deposition. Bed forms with their crest parallel to the flow are called longitudinal bed forms such as ribbons and ridges.

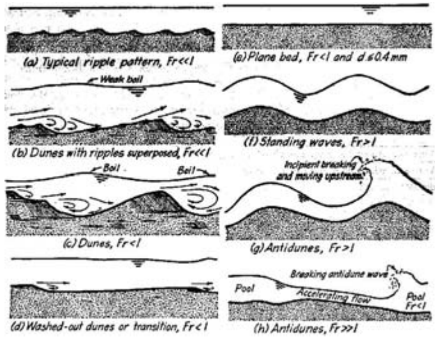
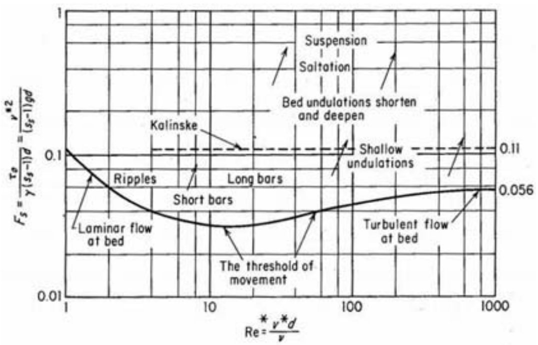



Figure 10.2: Bedforms and the Froude numbers at which they occur (after Richardson and Simons)

In the literature, various bed-form classification methods for sand beds are presented. The types of bed forms are described in terms of basic parameters (Froude number, suspension parameter, particle mobility parameter; dimensionless particle diameter).



The different bed forms were plotted by Shields on his diagram translated from the German on page 172 here into English:




Shields diagram from Henderson (1966, p413).



## Three-dimensional structures

Point bar at a river meander: the Cirque de la Madeleine in the Gorges de l'Ardèche, France. Photograph by Jean-Christophe Benoist.

The largest bed forms in the lower regime are sand bars (such as alternate bars, side bars, point bars, braid bars and transverse bars), which usually are generated in areas with relatively large transverse flow components (bends, confluences, expansions). Alternate bars are features with their crests near alternate banks of the river. Braid bars actually are alluvial "islands" which separate the anabranches of braided streams. Numerous bars can be observed distributed over the cross-sections. These bars have a marked streamwise elongation. Transverse bars are diagonal shoals of triangular-shaped plan along the bed. One side may be attached to the channel bank. These type of bars generally are generated in steep slope channels with a large width-depth ratio. The flow over transverse bars is sinuous (wavy) in plan. Side bars are bars connected to river banks in a meandering channel. There is no flow over the bar. The planform is roughly triangular. Special examples of side bars are point bars and scroll bars.