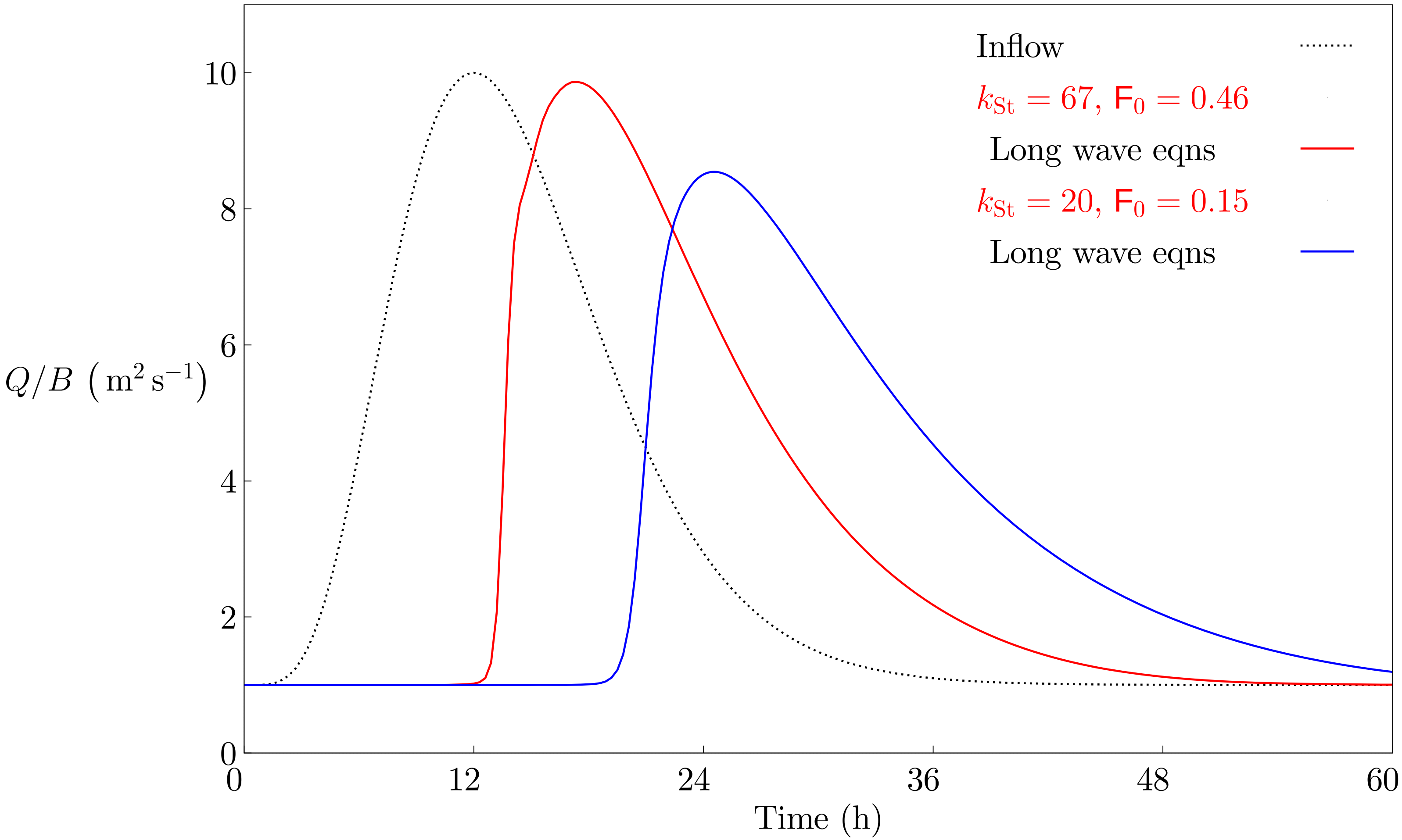
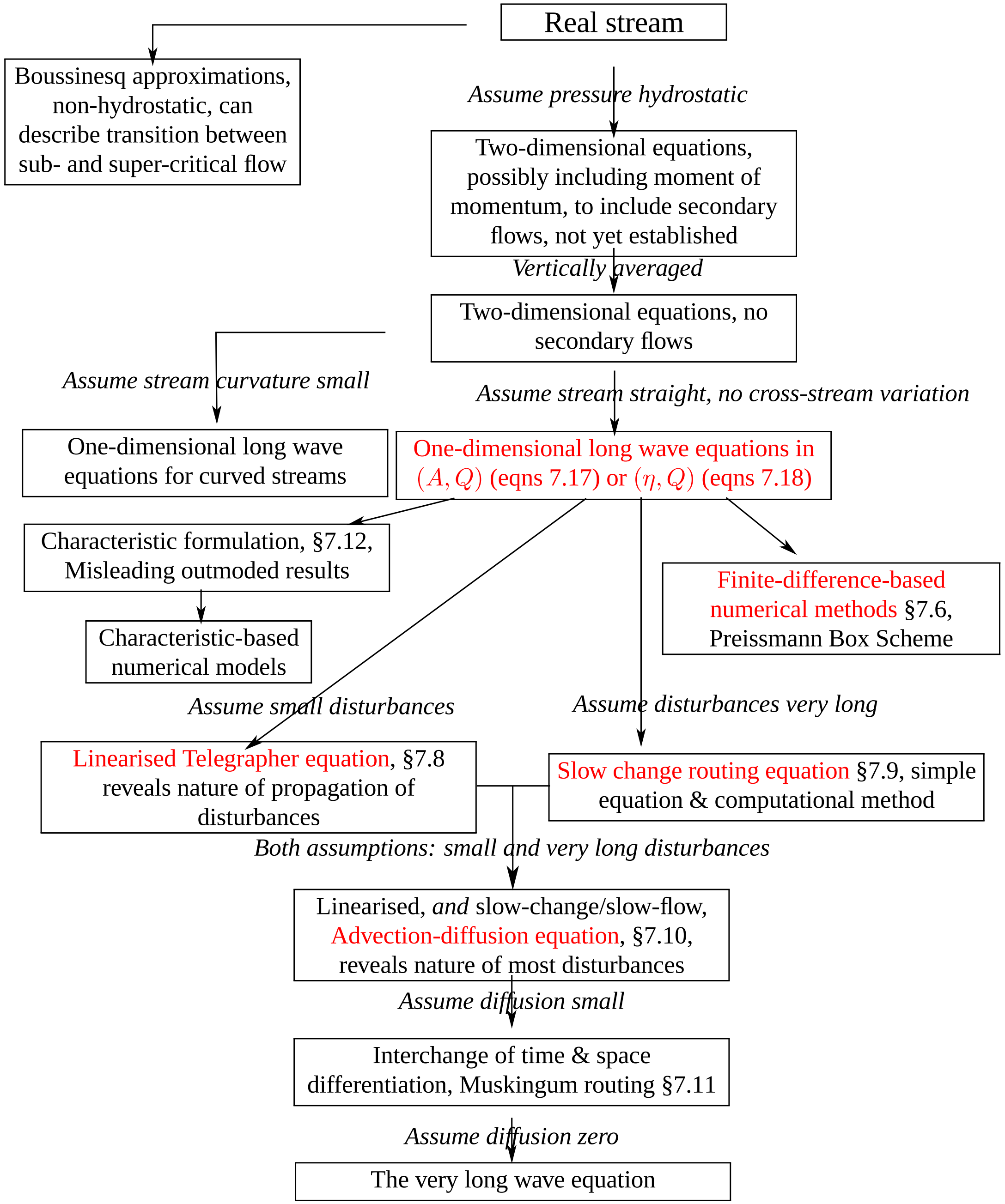
7.4 Examples of flood propagation



As an example we consider an infinitely-wide (no side friction) channel with a channel slope $S = 0.0005$ and length $50\ \mathrm{km}$. Two different boundary resistances were considered, Strickler $k_{St} = 67$ for a smooth boundary to give a large Froude number and $k_{St} = 20$ for a natural boundary.


Real stream
Boussinesq approximations, non-hydrostatic, can describe transition between sub- and super-critical flow
Assume pressure hydrostatic
Two-dimensional equations, possibly including moment of momentum, to include secondary flows, not yet established
Vertically averaged
Two-dimensional equations, no secondary flows
Assume stream curvature small
Assume stream straight, no cross-stream variation
One-dimensional long wave equations for curved streams
One-dimensional long wave equations in (A, Q) (eqns 7.17) or (η, Q) (eqns 7.18)
Characteristic formulation, §7.12, Misleading outmoded results
Characteristic-based numerical models
Finite-difference-based numerical methods §7.6, Preissmann Box Scheme
Assume small disturbances
Assume disturbances very long
Linearised Telegrapher equation, §7.8 reveals nature of propagation of disturbances
Slow change routing equation §7.9, simple equation & computational method
Both assumptions: small and very long disturbances
Linearised, and slow-change/slow-flow, Advection-diffusion equation, §7.10, reveals nature of most disturbances
Assume diffusion small
Interchange of time & space differentiation, Muskingum routing §7.11
Assume diffusion zero
The very long wave equation

7.6 Numerical solution of the long wave equations – FTQS scheme

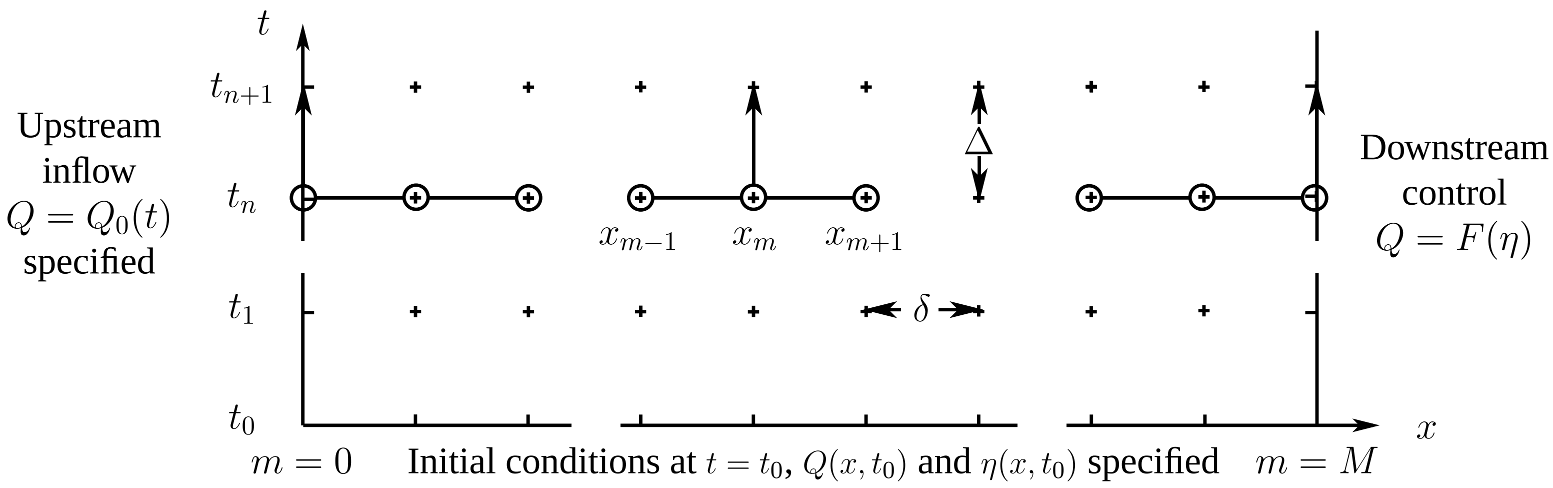


Figure 7.3: (x, t) axes showing computational grid, initial and boundary conditions, and three computational modules

We use a scheme where time derivatives are approximated using forward differences, and where x-derivatives are approximated using quadratic approximation, fitting a quadratic to three points, giving the **F**orward-**T**ime-**Q**uadratic-**S**pace scheme. The x-derivatives are

$$\left.\frac{\partial f}{\partial x}\right|_0 = \frac{-3f_0 + 4f_1 - f_2}{2\delta} + O(\delta) \;, \tag{7.19a}$$

$$\left.\frac{\partial f}{\partial x}\right|_m = \frac{f_{m+1} - f_{m-1}}{2\delta} + O(\delta) \,, \quad \text{for } m = 1, \ldots, M-1 \;, \tag{7.19b}$$

$$\left.\frac{\partial f}{\partial x}\right|_M = \frac{f_{M-2} - 4f_{M-1} + 3f_M}{2\delta} + O(\delta) \;. \tag{7.19c}$$

Using the obvious forward difference expressions for the time derivatives, the scheme applied to the (A, Q) formulation, equations (7.17), becomes

$$\frac{A_{m,n+1} - A_{m,n}}{\Delta} = i - \left.\frac{\partial Q}{\partial x}\right|_{m,n}, \tag{7.20a}$$

$$\frac{Q_{m,n+1} - Q_{m,n}}{\Delta} = -\frac{\partial}{\partial x}\left(\beta\frac{Q^2}{A}\right) - \frac{gA}{B}\frac{\partial A}{\partial x} + gA\tilde{S} - \Omega Q\left|Q\right|\bigg|_{m,n}. \tag{7.20b}$$

Both expressions are easily re-arranged to give explicit formulae for the terms in red, the values of A and Q at point m at the next time level $n+1$, each in terms of three values of A and three values of Q at the computational points at the previous time t_n, using one of the equations (7.19).

Liggett & Cunge (1975) claimed that the above scheme, the simplest and most obvious, was unconditionally unstable. This had some important implications, for it meant that the world was forced into using complicated schemes such as the Preissmann Box scheme, which form the basis of all commercial software. The lecturer (Fenton 2014) has discovered that their analysis is wrong, and that the scheme has a quite acceptable stability limitation, and it opens up the possibility for simpler computations of floods and flows in open channels. The Preissmann Box Scheme allows much larger time steps, but it is very complicated to apply.

7.7 Initial and boundary conditions

Initial conditions

Usually there is some initial flow in the channel which is constant if there is no inflow, $Q(x, t_0) = Q_0$. The next step is to determine the initial distribution of surface elevation η. The conventional method is to solve the Gradually-varied flow equation, using the equations and methods described in §8, as well as the downstream boundary condition, which is about to be described. A simpler method is to use the unsteady equations and computation scheme that will be used later anyway – simply start with an approximate solution for $\eta(x, t_0)$ (a straight line?) and let the unsteady dynamics take over, allowing disturbances to propagate downstream and out of the computational domain until the solution is steady. Then, for example, the main computation can be started.

Boundary conditions

Upstream

It is usually the upstream boundary condition that drives the whole model, where a flood or wave enters, via the specification of the time variation of $Q = Q(x_0, t)$ at the boundary. The surface elevation there is obtained as part of the computations. A common model inflow hydrograph is:

$$Q(x_0, t) = Q_{\text{min}} + (Q_{\text{max}} - Q_{\text{min}}) \left(\frac{t}{T_{\text{max}}} e^{1 - t/T_{\text{max}}} \right)^5,$$

where the event starts at $t = 0$ with Q_{min} and has a maximum Q_{max} at $t = T_{\text{max}}$.

We have two variables, however, Q and either A or η. To obtain this we just use the mass conservation FTQS expression (7.20a) to obtain the updated value of A or η at $m = 0$ at t_{n+1}. The equation of course applies up to and including the boundary point.

Downstream boundary - known stage-discharge relationship

- Where there is a downstream control structure such as a spillway, weir, gate, or flume, the stage-discharge relationship $Q(x_M, t) = F(\eta(x_M, t))$ must be known. For example, a weir might have a flow formula such as

$$Q = 0.6\sqrt{g}b\,(\eta - z_{\mathrm{c}})^{3/2}$$

  where b is the crest length and z_{c} is the elevation of the crest.

- We assume that the $Q = F(\eta)$ relationship is not affected by unsteadiness and non-uniformity, which probably holds for relatively short control structures mentioned

- A potential difficulty – we have one equation too many: we have the FTQS finite difference formulae based on mass conservation for $\eta(x_M, t_{n+1})$ and momentum conservation for $Q(x_M, t_{n+1})$ *and* the relation between Q and η

- However, a sudden change in section where a typical spillway, weir, gate, or flume is placed actually violates a fundamental assumption of the long wave momentum equation, that variation in the channel is long. We can easily ignore that equation near such a sudden change

- Fortunately, the mass conservation equation, is still valid near a sudden change – it requires only the assumption that water surface is horizontal across the channel.

- The procedure is: obtain the updated value $\eta(x_M, t_{n+1})$ from the FTQS finite difference formula for the *mass conservation* equation (7.20a), using values of Q at x_{M-2}, x_{M-1}, x_M and t_n and then use the *stage-discharge relationship* to calculate $Q(x_M, t_{n+1}) = F(\eta(x_M, t_{n+1}))$

Open downstream boundary

- A common boundary is where the computational domain is artificially truncated at some point in the stream. This is sometimes called *Normal Depth* boundary and standard practice is that the computational domain be artificially extended and this boundary condition be used far enough downstream from the study area that it does not affect the results there.
- The lecturer prefers a different approach, and this is simply to treat the boundary as if it were just any other part of the river (which it is!) and to use both long wave equations to update both η and Q there, calculating the necessary derivatives $\partial\eta/\partial x$ and $\partial Q/\partial x$ from upstream finite difference formulae and simply treating the end point as if it were an ordinary point in the stream and using *both* FTQS formulae for $\eta\left(x_M, t_{n+1}\right)$ and $Q\left(x_M, t_{n+1}\right)$ there, with the three-point leftwards approximations for the last point x_M in terms of values at x_{M-2}, x_{M-1}, and x_M.
- This works very well in practice.