# 9. Hydrometry - measurement and analysis

## 9.1 Definitions

In English, the traditional word used to describe the measurement of water levels and flow volumes is “Hydrography”. That is ambiguous, for that word is also used for the measurement of water depths for navigation purposes, has been so used since the great navigators of the eighteenth century. Organisations with names like “National Hydrographic Service” are usually only concerned with the mapping of an area of sea and surrounding coastal detail.

Here we follow Boiten (2000) and Morgenschweis (2010) who provide a refreshingly modern approach to the topic, calling it “Hydrometry”, the “measurement of water”. In these notes, a practitioner will be called a hydrometrician, but the term hydrograph will be retained for a record, either digital or graphical, of the variation of water level or flow rate with time.

Two modern documents from the World Meteorological Organization provide more background. Experimental techniques are described in WMO (2010*a*), and methods of analysis in WMO (2010*b*). It is remarkable, however, that a field so important has received little benefit from hydraulics research.

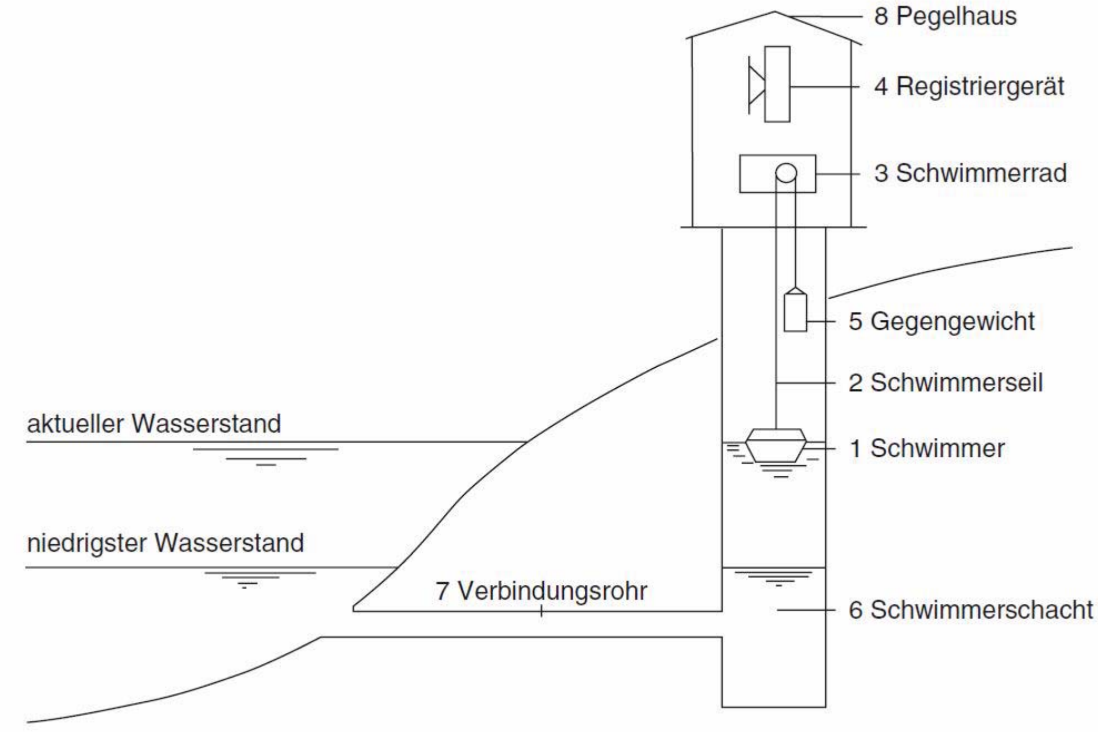
## 9.2 The Problem

Almost universally the routine measurement of the state of a river is that of the stage, the surface elevation at a gauging station. While that is an important quantity in determining the danger of flooding, another important quantity is the actual volume flow rate past the gauging station. Accurate knowledge of this instantaneous discharge - and its time integral, the total volume of flow - is crucial to many hydrologic investigations and to practical operations of a river and its chief environmental and commercial resource, its water. Examples include decisions on the allocation of water resources, the design of reservoirs and their associated spillways, the calibration of models, and the interaction with other computational components of a network.

Stage is usually simply measured. Measuring the flow rate, the discharge, is rather more difficult. Almost universally, occasionally (once per month, or more likely, once per year) it is obtained by measuring the velocity field in detail and integrating it with respect to area. At the same time, the water level is measured. This gives a pair of values $(\eta_i, Q_i)$ which obtained on that day. Over a long period, a finite number of such data pairs are obtained using this laborious method. A curve that approximates those points is calculated, to give a function $Q_{\mathrm{r}}(\eta)$, a *Rating Curve*.

Separately, the actual stage can be measured easily and monitored almost continuously at any time, and automatically transmitted and recorded at intervals of one hour or one day, to give a *Stage Hydrograph*, a discrete representation of $\eta_n = \eta(t_n), n = 0, 1, \ldots$. To get the corresponding *Discharge Hydrograph*, each value of $\eta_n$ is considered and from the rating curve the corresponding $Q_n = Q_{\mathrm{r}}(\eta_n), n = 0, 1, \ldots$ are calculated. Values of $\eta_n$ and $Q_n$ are published and made available.

# 9.3 Routine measurement of water levels




Stilling well & level recorder (Morgenschweis 2010)

Most water level gauging stations are equipped with a sensor or gauge plus a recorder. In many cases the water level is measured in a stilling well, thus eliminating strong oscillations.

**Staff gauge:**

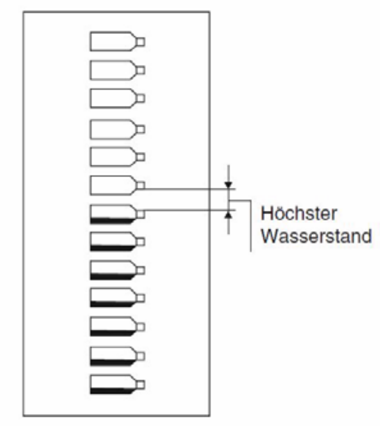
This is the simplest type, with a graduated gauge plate fixed to a stable structure such as a pile, bridge pier, or a wall. Where the range of water levels exceeds the capacity of a single gauge, additional ones may be placed on the line of the cross section normal to the plane of flow.

**Float gauge:**

A float inside a stilling well, connected to the river by an inlet pipe, is moved up and down by the water level. Fluctuations caused by short waves are almost eliminated. The movement of the float is transmitted by a wire passing over a float wheel, which records the motion, leading down to a counterweight.

**Pressure transducers:**

Water level is measured as hydrostatic pressure and transformed into an electrical signal *via* a semi-conductor sensor. These are best suited for measuring water levels in open water (the effect of short waves dies out almost completely within half a wavelength down into the water). They should compensate for changes in the atmospheric pressure, and if air-vented cables cannot be provided air pressure must be measured separately.




**Peak level indicators:**

There are some indicators of the maximum level reached by a flood, such as arrays of bottles which tip and fill when the water reaches them, or a staff coated with soluble paint.

**Bubble gauge:**

This is based on measurement of the pressure which is needed to produce bubbles through an underwater outlet. These are used at sites where it would be difficult to install a float-operated recorder or pressure transducer. From a pressurised gas cylinder or small compressor gas is led along a tube to some point under the water (which will remain so for all water levels) and small bubbles constantly flow out through the orifice. The pressure in the measuring tube corresponds to that in the water above the orifice. Wind waves should not affect this.

**Ultrasonic sensor:**

These are used for continuous non-contact level measurements in open channels. The sensor points vertically down towards the water and emits ultrasonic pulses at a certain frequency. The inaudible sound waves are reflected by the water surface and received by the sensor. The round trip time is measured electronically and appears as an output signal proportional to the level. A temperature probe compensates for variations in the speed of sound in air. They are accurate but susceptible to wind waves.

## 9.4 Occasional measurement of discharge

Traditional manner of taking current meter readings. In deeper water a boat is used.

Most methods of measuring the rate of volume flow past a point are single measurement methods which are not designed for routine operation. Below, some will be described that are methods of continuous measurements.

### Velocity area method (“current meter method”)

The area of cross-section is determined from soundings, and flow velocities are measured using propeller current meters, electromagnetic sensors, or floats. The mean flow velocity is deduced from points distributed systematically over the river cross-section. In fact, what this usually means is that two or more velocity measurements are made on each of a number of vertical lines, and any one of several empirical expressions used to calculate the mean velocity on each vertical, the lot then being integrated across the channel.

Calculating the discharge requires integrating the velocity data over the whole channel - what is required is the area integral of the velocity, that is $Q = \int u \, dA$. If we express this as a double integral we can write

$$Q = \int_B \int_{Z(y)}^{Z(y)+h(y)} u \, dz \, dy, \tag{9.1}$$

so that we must first integrate the velocity from the bed $z = Z(y)$ to the surface $z = Z(y) + h(y)$, where $h$ is the local depth. Then we have to integrate these contributions across the channel, for values of the transverse co-ordinate $y$ over the breadth $B$.

**Calculation of mean velocity in the vertical**

The first step is to compute the integral of velocity with depth, which hydrometricians think of as calculating the mean velocity over the depth. Consider the law for turbulent flow over a rough bed:

$$u = \frac{u_*}{\kappa} \ln \frac{z - Z}{z_0}, \tag{9.2}$$

where $u_*$ is the shear velocity, $\kappa = 0.4$, $\ln()$ is the natural logarithm to the base $e$, $z$ is the elevation above the bed, and $z_0$ is the elevation at which the velocity is zero. (It is a mathematical artifact that below this point the velocity is actually negative and indeed infinite when $z = 0$ – this does not usually matter in practice). If we integrate equation (9.2) over the depth $h$ we obtain the expression for the mean velocity:

$$\bar{u} = \frac{1}{h} \int_{Z(y)}^{Z(y)+h(y)} u\, dz = \frac{u_*}{\kappa} \left( \ln \frac{h}{z_0} - 1 \right). \tag{9.3}$$

Now it is assumed that two velocity readings are made, obtaining $u_1$ at $z_1$ and $u_2$ at $z_2$. This gives enough information to obtain the two quantities $u_*/\kappa$ and $z_0$. Substituting the values for point 1 into equation (9.2) gives us one equation and the values for point 2 gives us another equation. Both can be solved to give the simple formula for the mean velocity in terms of the readings at the two points:

$$\bar{u} = \frac{u_1 \left(\ln(z_2/h)+1\right) - u_2 \left(\ln(z_1/h)+1\right)}{\ln\left(z_2/z_1\right)}. \tag{9.4}$$

As it is probably more convenient to measure and record depths rather than elevations above the bottom, let $h_1 = h - z_1$ and $h_2 = h - z_2$ be the depths of the two points, when equation (9.4) becomes

$$\bar{u} = \frac{u_1 \left(\ln(1 - h_2/h)+1\right) - u_2 \left(\ln(1 - h_1/h)+1\right)}{\ln\left(\left(h - h_2\right) / \left(h - h_1\right)\right)}. \tag{9.5}$$

This expression gives the freedom to take the velocity readings at any two points. This would simplify streamgauging operations, for it means that the hydrometrician, after measuring the depth $h$, does not have to calculate the values of $0.2h$ and $0.8h$ and then set the meter at those points, as is done in current practice. Instead, the meter can be set at any two points, within reason, the depth and the velocity simply recorded for each, and equation (9.5) applied. This could be done either *in* *situ* or later when the results are being processed. This has the potential to speed up hydrographic measurements.

If the hydrometrician were to use the traditional two points, then setting $h_1 = 0.2h$ and $h_2 = 0.8h$ in equation (9.5) gives the result

$$\bar{u} = 0.4396\, u_{0.2h} + 0.5604\, u_{0.8h} \approx 0.44 u_{0.2h} + 0.56 u_{0.8h}\,, \tag{9.6}$$

whereas the conventional hydrographic expression is

$$\overline{u} = \tfrac{1}{2}\left(u_{0.2h} + u_{0.8h}\right), \tag{9.7}$$

that is, the mean of the readings at 0.2 of the depth and 0.8 of the depth. The nominally more accurate expression is just as simple as the traditional expression in a computer age, yet is based on an exact analytical integration of the equation for a turbulent boundary layer.

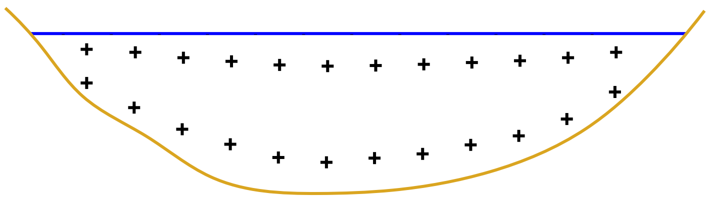


Figure 9.1: Cross-section of stream, showing velocity measurement points

The formula (9.6) has been tested by taking a set of gauging results. A canal had a maximum depth of 2.6m and was 28m wide, and a number of verticals were used. The conventional formula (9.7), the mean of the two velocities, was accurate to within 2% of equation (9.6) over the whole range of the readings, with a mean difference of 1%. That error was always an overestimate. The more accurate formula (9.5) is hardly more complicated than the traditional one, and it should in general be preferred. Although the gain in accuracy was slight in this example, in principle it is desirable to use an expression which makes no numerical approximations to that which it is purporting to evaluate. This does not necessarily mean that either (9.7) or (9.6) gives an accurate integration of the velocities which were encountered in the field. In fact, one complication is where, as often happens in practice, the velocity distribution near the surface actually bends back such that the maximum velocity is below the surface.

**Integration of the mean velocities across the channel**

The problem now is to integrate the readings for mean velocity at each station across the width of the channel. Here traditional standards commit an error – often the *Mean-Section* method is used. In this the mean velocity between two verticals is calculated and then this multiplied by the area between them, so that, given two verticals $i$ and $i+1$ separated by $b_i$ the expression for the contribution to discharge is assumed to be

$$\delta Q_i = \frac{1}{4} b_i \left(h_i + h_{i+1}\right)\left(\overline{u}_i + \overline{u}_{i+1}\right).$$

This is not correct. From equation (9.1), the task is actually to integrate across the channel the quantity which is the mean velocity times the depth. For that the simplest expression is the Trapezoidal rule:

$$\delta Q_i = \frac{1}{2} b_i \left(\overline{u}_{i+1}\, h_{i+1} + \overline{u}_i\, h_i\right)$$

To examine where the Mean-Section Method is worst, we consider the case at one side of the channel, where the area is a triangle. We let the water's edge be $i=0$ and the first internal point be

$i = 1$, then the Mean-Section Method gives

$$\delta Q_0 = \frac{1}{4} b_0 \overline{u}_1 h_1,$$

while the Trapezoidal rule gives

$$\delta Q_0 = \frac{1}{2} b_0 \overline{u}_1 h_1,$$

which is correct, and we see that the Mean-Section Method computes only half of the actual contribution. The same happens at the other side. Contributions at these edges are not large, and in the middle of the channel the formula is not so much in error, but in principle the Mean-Section Method is wrong and should not be used. Rather, the Trapezoidal rule should be used, which is just as easily implemented. In a gauging in which the lecturer participated, a flow of $19.60\,\mathrm{m^3 s^{-1}}$ was calculated using the Mean-Section Method. Using the Trapezoidal rule, the flow calculated was $19.92\,\mathrm{m^3 s^{-1}}$, a difference of 1.6%. Although the difference was not great, practitioners should be discouraged from using a formula which is wrong.

**An alternative global "spectral"approach with least-squares fitting**

It is strange that only very local methods are used in determining the vertical velocity distribution. Here we consider a significant generalisation, where we consider velocity distributions given by a more general law, assuming an additional linear and an additional quadratic term in the velocity profile:

$$u(y,z) = a_0(y) \ln \frac{z - Z(y)}{z_0} + a_1(y)\; z + a_2(y)\, z^2, \tag{9.8}$$

but where the coefficients $a_0(y)$, $a_1(y)$, and $a_2(y)$ are actually polynomials in the transverse co-ordinate. The whole expression is a global function, that approximates the velocity over the whole section. If the polynomials in $y$ each have $J$ terms, then the total number of unknown coefficients is $3J$. Consider a number of flow measurements $U_n$ for $n = 1$ to $N$, where we presume that the corresponding bed elevation is also measured, however that is not essential to the method. We compute the total sum of the errors squared, using equation (9.8):

$$\varepsilon = \sum_{n=1}^{N} \left(u\left(y_n, z_n\right) - U_n\right)^2 ,$$

and we use package software to find the coefficients such that the total error $\varepsilon$ is minimised. Or, as one says, the function is "fitted to the data". This method does not require points to be in vertical lines, although it is often convenient to measure points like that, as well as the corresponding bed level. An example of the results is given in figure 9.2, where more than two points were used on each vertical line. It can be seen that results are good - and we see an import feature that the conventional method ignores – almost everywhere there is a velocity maximum in the vertical.

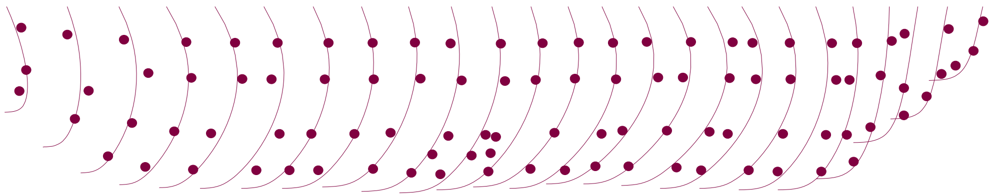


Figure 9.2: Cross-section of canal with velocity profiles and data points plotted transversely, showing fit by global function

## Dilution methods

In channels where cross-sectional areas are difficult to determine (*e.g.* steep mountain streams) or where flow velocities are too high to be measured by current meters *dilution* or *tracer* methods can be used, where continuity of the tracer material is used with steady flow. The rate of input of tracer is measured, and downstream, after total mixing, the concentration is measured. The discharge in the stream immediately follows.

## Ultrasonic flow measurement

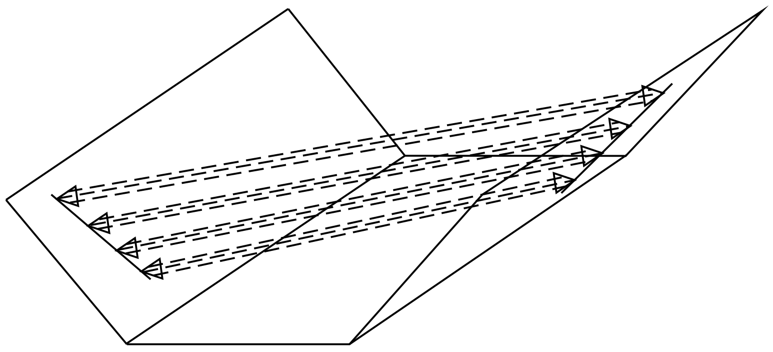


Figure 9.3: Array of four ultrasonic beams in a channel

In this case, sound generators are placed along the side of a channel and beamed so as to cross it diagonally. They are reflected on the other side, and the total time of travel of the sound waves are measured. From that it is possible to calculate the mean water velocity along the channel – the sound "samples" the water velocity at all points. Then, to get the total discharge it is necessary to integrate the mean velocity of the paths in the vertical. This, unfortunately, is where the story ends unhappily. The performance of the trade and scientific literature has been poor. Several trade brochures advocate the routine use of a single beam, or maybe two, suggesting that that is adequate (see, for example, Boiten 2000, p141). In fact, with high-quality data for the mean velocity at two or three levels, there is no reason not to use accurate integration formulae. However, practice in this area has been quite poor, as trade brochures that the author has seen use the inaccurate Mean-Section Method for integrating vertically over only three or four data points, when its errors would be rather larger than when it is used for many verticals across a channel, as described previously. The lecturer found that no-one wanted to know of his discovery.

## Acoustic-Doppler Current Profiling (ADCP) methods

In these, a beam of sound of a known frequency is transmitted into the fluid, often from a boat. When the sound strikes moving particles or regions of density difference moving at a certain speed, the sound is reflected back and received by a sensor mounted beside the transmitter. According to the Doppler effect, the difference in frequency between the transmitted and received waves is a direct measurement of velocity. In practice there are many particles in the fluid and the greater the area of flow moving at a particular velocity, the greater the number of reflections with that frequency shift. Potentially this method is very accurate, as it purports to be able to obtain the velocity over quite small regions and integrate them up. However, this method does not measure in the top 15% of the depth or near the boundaries, and the assumption that it is possible to extract detailed velocity profile data from a signal seems to be optimistic. The lecturer remains unconvinced that this method is as accurate as is claimed.

# Electromagnetic methods

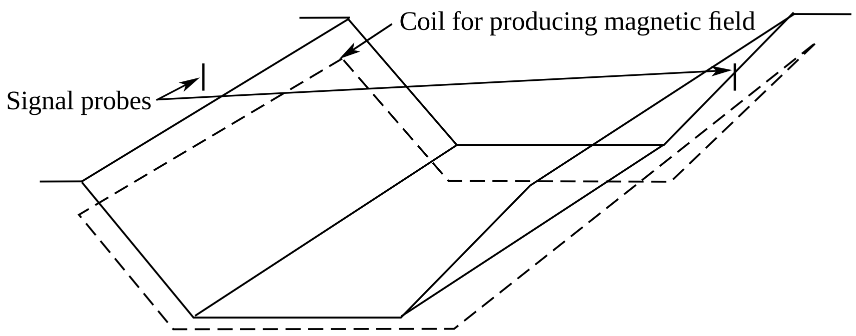



Figure 9.4: Electromagnetic installation, showing coil and signal probes

The motion of water flowing in an open channel cuts a vertical magnetic field which is generated using a large coil buried beneath the river bed, through which an electric current is driven. An electromotive force is induced in the water and measured by signal probes at each side of the channel. This very small voltage is directly proportional to the average velocity of flow in the cross-section. This is particularly suited to measurement of effluent, water in treatment works, and in power stations, where the channel is rectangular and made of concrete; as well as in situations where there is much weed growth, or high sediment concentrations, unstable bed conditions, backwater effects, or reverse flow. This has the advantage that it is an integrating method, however in the end recourse has to be made to empirical relationships between the measured electrical quantities and the flow.

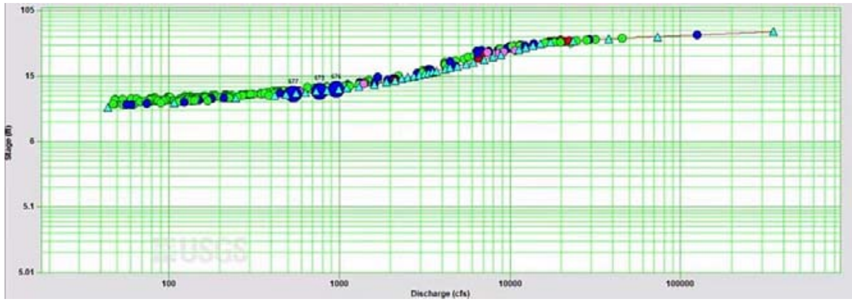
# 9.5 Rating curves – the analysis and use of stage and discharge measurements

## The state of the art – the power function

The generation of rating curves from data is a problem that is of crucial importance, but has had little research attention and is done very badly all around the world. Almost universally it is believed that they *must* follow a power function

$$Q = C\,(h - h_0)^{\mu}\,, \tag{9.9}$$

where $h$ is water surface elevation (stage), $C$ is a constant, $h_0$ is a constant elevation reference level, for zero flow, and $\mu$ is a constant with a typical value in the range $1.5$ to $2.5$.




US Geological Survey - fitting three straight lines to data segments

If we take logarithms of both sides of equation (9.9),

$$\log Q = \log C + \mu \log\,(h - h_0)\,, \tag{9.10}$$

then on a figure plotting $\log Q$ against $\log\,(h - h_0)$, a straight line is obtained. Often in practice, the data is divided and different straight-line approximations are used, as in the figure.

A problem is that $h_0$, the nominal zero flow point, is not initially known and has to be found, which is actually a difficult nonlinear problem. A larger problem is that the equation is only a rough approximation, but because of its ability occasionally to describe roughly almost all of a rating curve, it has acquired an almost-sacred status, and far too much attention has been devoted to it rather than addressing the problems of how to approximate rating data generally and accurately. Much modelling and computer software follow its dictates. It really has been believed to be a "law".

The reason that the power law has been believed to be so powerful (sorry) is that hydrologists believe the hydraulic formula – and it does describe the discharge of a sharp-crested infinitely-wide weir in water of infinite depth and the steady uniform flow in an infinitely-wide channel. We know enough hydraulics to know that not all rivers satisfy those conditions ... and we treat the problem as one, not of hydraulics, but of *data approximation*, because the hydraulics are complicated.

# The hydraulics of a gauging station

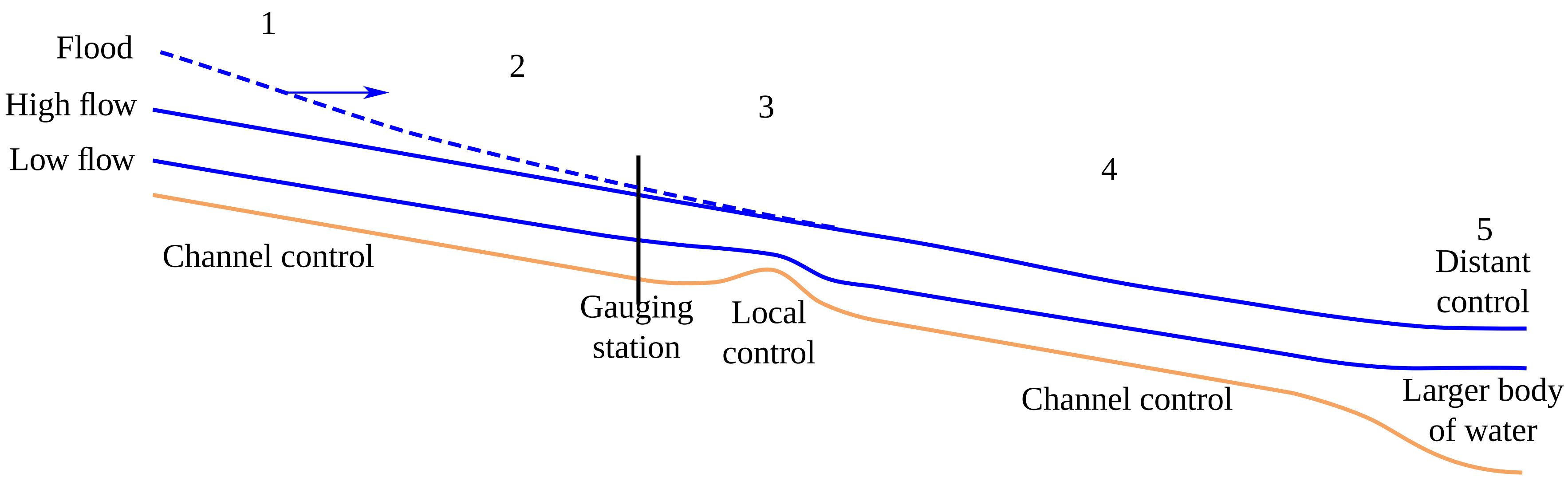



Figure 9.5: Section of river showing different controls at different water levels with implications for the stage discharge relationship at the gauging station shown

**Local control:** Just downstream of the gauging station is often some sort of fixed control which may be some local topography such as a rock ledge which means that for relatively small flows there is a definite relationship between the head over the control and the discharge, similar to a weir. This will control the flow for small flows.

**Channel control:** For larger flows the effect of the fixed control is to ”drown out”, to become unimportant, and where the control is due to resistance in the channel.

**Overbank control:** For larger flows when the river breaks out of the main channel and spreads onto the surrounding floodplain, the control is also due to resistance, but where the geometry of channel and nature of the resistance is different.

**Distant control:** There may be something such as the larger river downstream shown as a distant control in the figure. In our work on Steady Flow, we saw that backwater influence can extend for a long way.

In practice, the natures of the controls are unknown.

## Data approximation

The global representation of $Q$ by a polynomial has been in the background for some time:

$$Q = a_0 + a_1 h + a_2 h^2 + \ldots + a_M h^M = \sum_{m=0}^{M} a_m h^m \,, \tag{9.11}$$

where $a_0, a_1, \ldots, a_M$ are coefficients. Standard linear least-squares methods can be used to determine the coefficients, but it has never really succeeded.

Fenton & Keller (2001, §6.3.2) suggested writing the polynomial for $Q$ raised to the power $\nu$, specified *a priori*:

$$Q^\nu = a_0 + a_1 h + a_2 h^2 + \ldots + a_M h^M = \sum_{m=0}^{M} a_m h^m \,, \tag{9.12}$$

which is a simple generalisation of the power function to $Q = \left(a_0 + a_1 h + a_2 h^2 + \ldots\right)^{1/\nu}$. A value of $\nu = \frac{1}{2}$ was recommended as that was the mean value in the hydraulic discharge formulae for a sequence of weir and channel cross-sections that modelled local and channel control.

The use of such a fractional value has two effects:

1. For small flows, $h$ small, the data usually is such that

$$Q^{\nu} = a_0 + a_1 h, \tag{9.13}$$

   that is

$$Q = (a_0 + a_1 h)^{1/\nu} = C\,(h - h_0)^{\mu}\,,$$

   so it looks like the simple power law! In fact, $\nu = \frac{1}{2}$ is a good approximation. In that low-flow limit the polynomial just has to simulate nearly-linear variation, which it can easily do.

2. For large flows, the use of $Q^{1/2}$ means that the magnitude of the dependent variable to be approximated is much smaller, so that, instead of a range of say, $Q = 1$ to $10^4\,\mathrm{m}^3\mathrm{s}^{-1}$, a numerical range $1$ to $10^2$ has to be approximated.

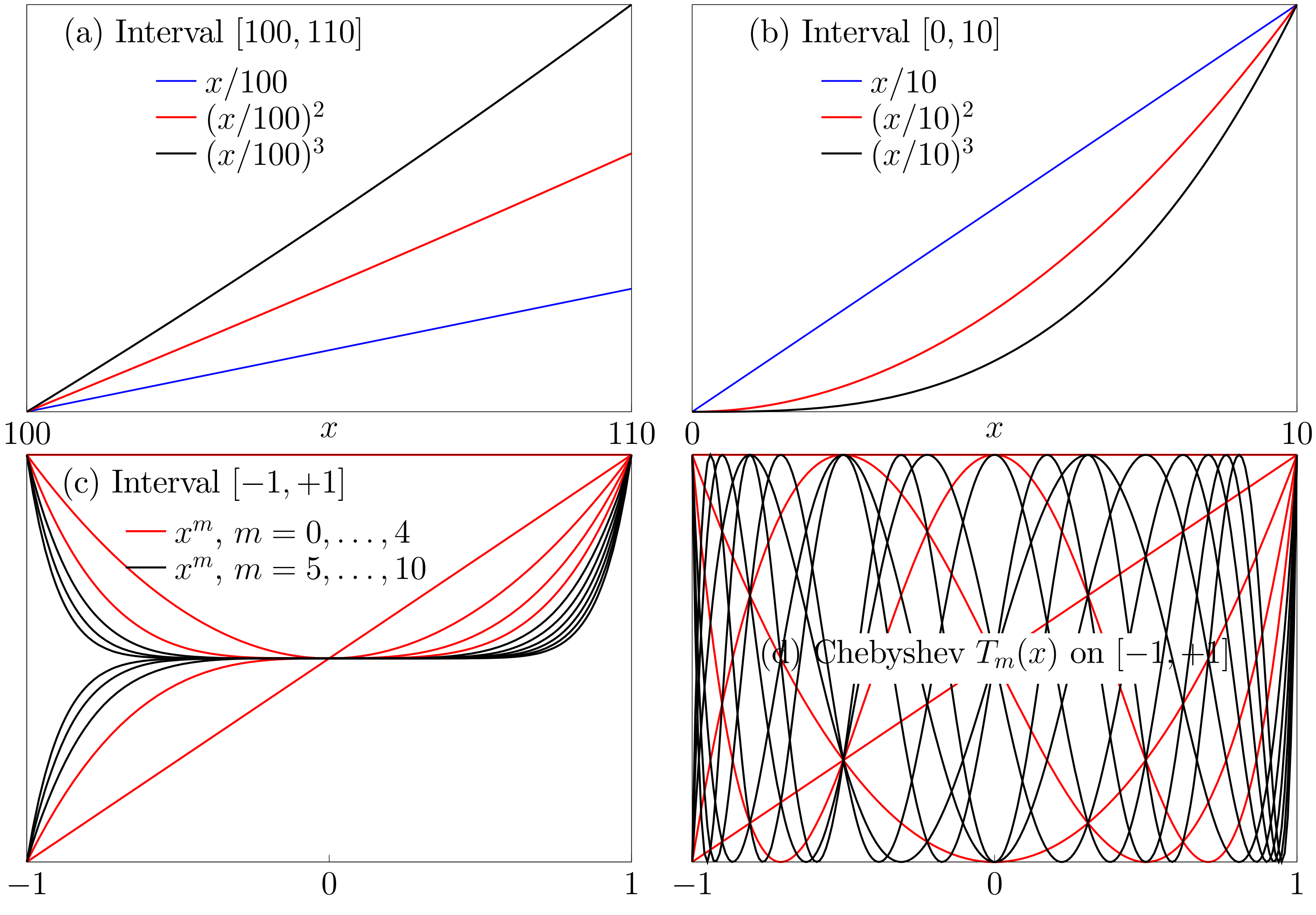
The lecturer (Fenton 2015) has shown it is better to generalise equation (9.12) by considering the approximating function to be made up, not of monomials $h^m$, but more general Chebyshev functions

$$Q^{\nu} = \sum_{m=0}^{M} a_m T_m\,(y)\ \ . \tag{9.14}$$

With these modifications, global approximation is more stable and accurate.

# Problems with global interpolation and approximation

The simplest set of basis functions are the *monomials* $p_m(x) = x^m$. They are not very good, as they all look rather like each other for large $x$ and for $m = 2$ or greater. Individual basis functions $p_m(x)$ should look different from each other so that irregular variation can be described efficiently.



# Least-squares approximation

The coefficients $a_m$ can be obtained by least-squares software, minimising the sum of the weighted squares of the errors of the approximation over $N$ data points,

$$\varepsilon_2 = \sum_{n=1}^{N} w_n \left( \sum_{m=0}^{M} a_m \, T_m \left( y_n \right) - Q_n^{\nu} \right)^2 ,$$

where the $y_n$ are obtained from the $h_n$ by scaling the range of all stage measurements $[h_{\min}, h_{\max}]$. The $w_n$ are the weights for each rating point, giving the freedom to weight some points more if one wanted the rating curve to approximate them more closely, or they could be set to be a decaying function of the age of the data point, so that the effects of changes with time could be examined. Or, a less-trusted data point could be given a smaller weight. Often, however, all the $w_n$ will be 1.

# An example

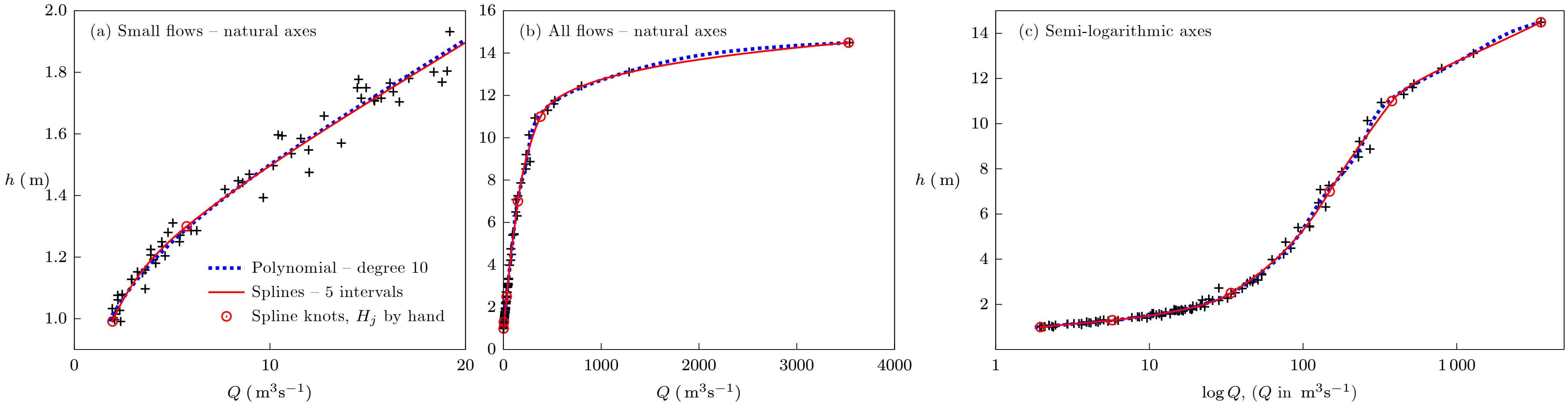



Figure 9.6: Noxubee River near Geiger, AL, USA, USGS Station 02448500, 1970s

- The example has quite a striking ideal form showing local, channel and overbank control.
- Variation at the low flow end can be rapid, and can have a vertical gradient and high curvature.
- The discharge may extend over 3 or 4 orders of magnitude (factor of $1000$ or $10000$)
- There can be rapid variation between these different regimes
- There are two curves that have been fitted by least-squares methods, one using Chebyshev polynomials, the other using local spline approximation. Both have worked well.

## Possible problems

There are several problems associated with the use of a Rating Curve:

- Discharge is rarely measured during a flood, and the quality of data at the high flow end of the curve might be quite poor.
- There are a number of factors which might cause the rating curve not to give the actual discharge, some of which will vary with time. Factors affecting the rating curve include:
  – The channel changing as a result of modification due to dredging, bridge construction, or vegetation growth.
  – Sediment transport - where the bed is in motion, which can have an effect over a single flood event, because the effective bed roughness can change during the event. As a flood increases, any bed forms present will tend to become larger and increase the effective roughness, so that friction is greater after the flood peak than before, so that the corresponding discharge for a given stage height will be less after the peak.
  – Backwater effects - changes in the conditions downstream such as the construction of a dam or flooding in the next waterway.
  – Unsteadiness - in general the discharge will change rapidly during a flood, and the slope of the water surface will be different from that for a constant stage, depending on whether the discharge is increasing or decreasing, also contributing to a flood event appearing as a loop on a stage-discharge diagram.
  – Variable channel storage - where the stream overflows onto flood plains during high discharges, giving rise to different slopes and to unsteadiness effects.

## Modelling rating curve changes with time

The importance of each data point can be weighted according to their age, so that the oldest points have the smallest contributions to the least squares error, and the most recent gaugings can be rationally incorporated to give the most recent rating curve. In fact, the rating curve can be constructed for any day, now or in the past.

We use a smooth function, decaying into the past, to keep all the points to some extent in determining the shape of the curve, the exponential weight factor $w(\tau) = \exp(-\alpha\tau)$, where $\alpha$ is a decay constant. Writing $\tau_{1/2}$ for the "half-life", the age at which the weight decays by a factor of $\frac{1}{2}$, then the expression becomes

$$w(t_0 - t_n) = \left(\frac{1}{2}\right)^{(t_0 - t_n)/\tau_{1/2}},$$

where $t_0$ is the date for which the rating curve is required, and $t_n$ is the date when point $n$ was established. If $t_n > t_0$ a value of zero is used. This was applied to 31 years of data from USGS Station 02448500 on the Noxubee River near Geiger, AL, USA, shown in Fig. 9.7. The approximating spline method with 6 hand-allocated intervals was used, with a $Q^{1/2}$ fit. It can be seen how the rating curve, and presumably the bed, has moved down over the 31 years.

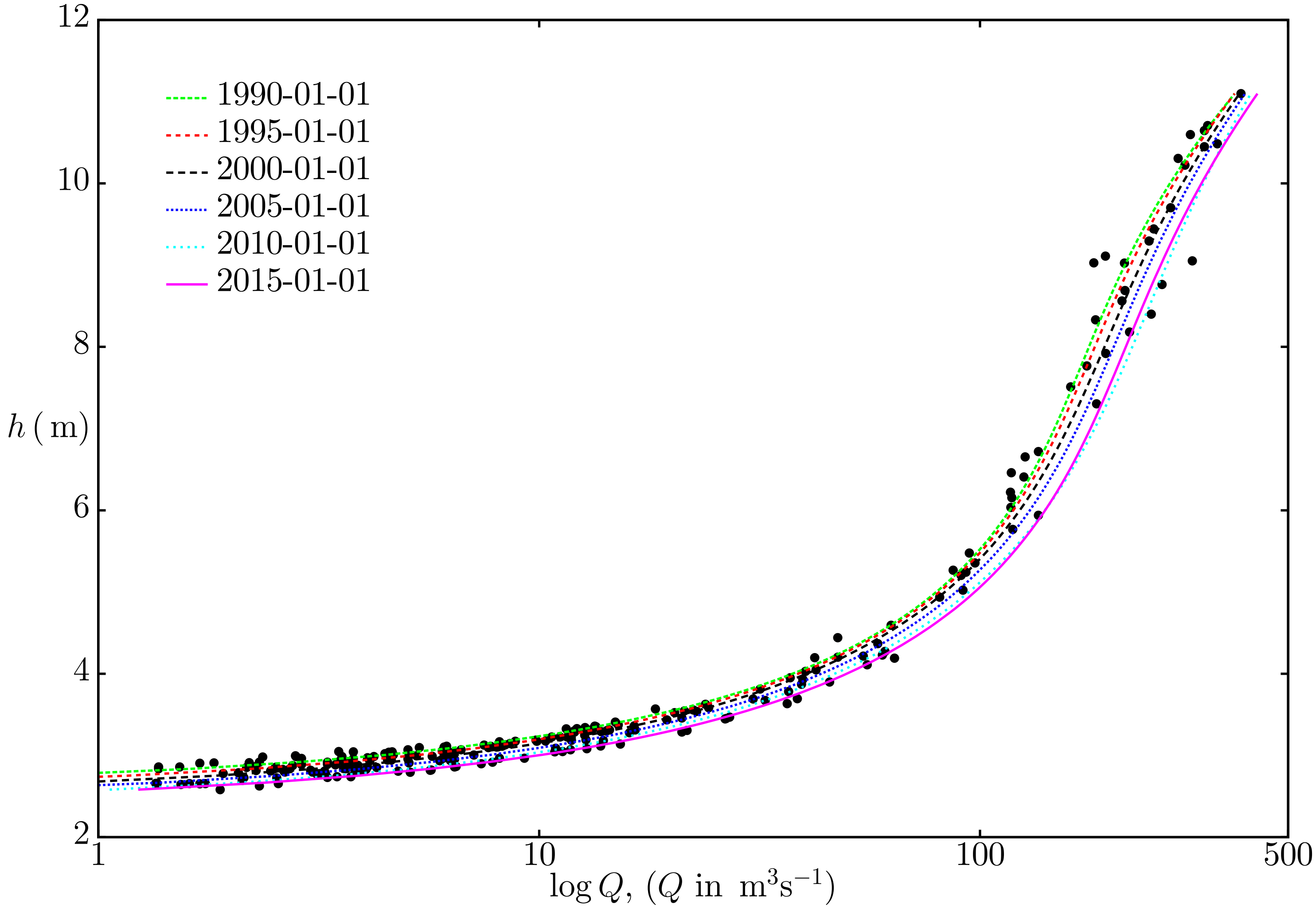



Figure 9.7: Calculation of rating curves on specific days using weights that are a function of measurement age — Noxubee River near Geiger, AL, USA, USGS Station 02448500, from 1984-10-02 to 2015-05-11

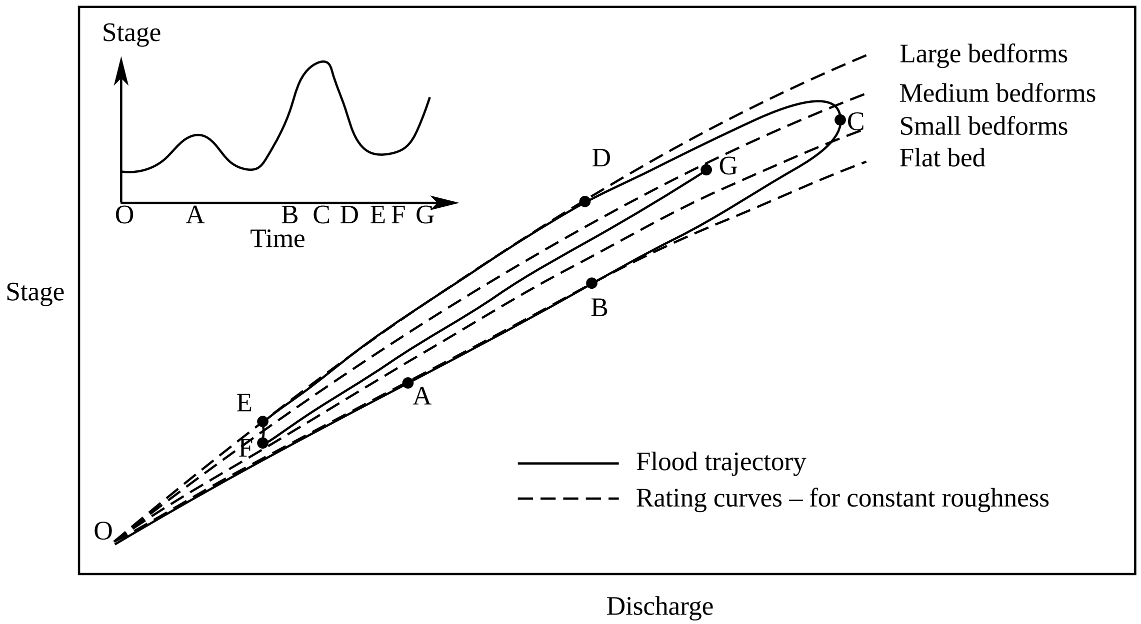
# The effects of bed roughness and bed changes on rating curves in alluvial streams

If there is a rapidly-changing flow event such as a flood, roughness and hence resistance might also change relatively quickly, and the relationship between stage and discharge changes with time such that if we were able to measure and plot it throughout the unsteady event we would obtain a looped curve with two discharges for each stage, and *vice versa*, before and after the flow maximum. This is usually described as a *looped rating curve*. The lecturer has usually been sceptical of that term, viewing it more as a *looped flow trajectory on rating axes*.

Let us consider the mechanism by which changes in resistance cause the flood trajectories to be looped, by considering a hypothetical and idealised situation. We do not know how much bed-forms and how much individual grains are responsible for most resistance.

The figure on the next page is plotted with rating curve axes, stage versus discharge. The rating curves which would apply if the resistance were a particular value are shown, for a flat bed with co-planar grains, and for various increasing resistances.

In the top left corner is a stage-time graph with two flood events, and another not yet completed. The points labelled O, A, ..., G are also shown on the *flow trajectory*, showing the actual relationship between stage and discharge at each time.




O: flow is low, over a flat co-planar bed after a period of steady flow.

A: the flow increases. The flow is not enough to change the nature of the bed, and the flood trajectory follows the flat-bed rating curve up to here.

B: the bed is no longer stable, grains move and bed forms develop. Accordingly, the resistance is greater and the stage increases.

C: flood peak has arrived, resistance continues to increase, a little later the stage is a maximum.

D: resistance and bedforms have continued to grow until here, although flow is decreasing.

E: the flow has decreased much more quickly than the bed can adjust, and the point is close to the instantaneous rating curve corresponding to the greatest resistance.

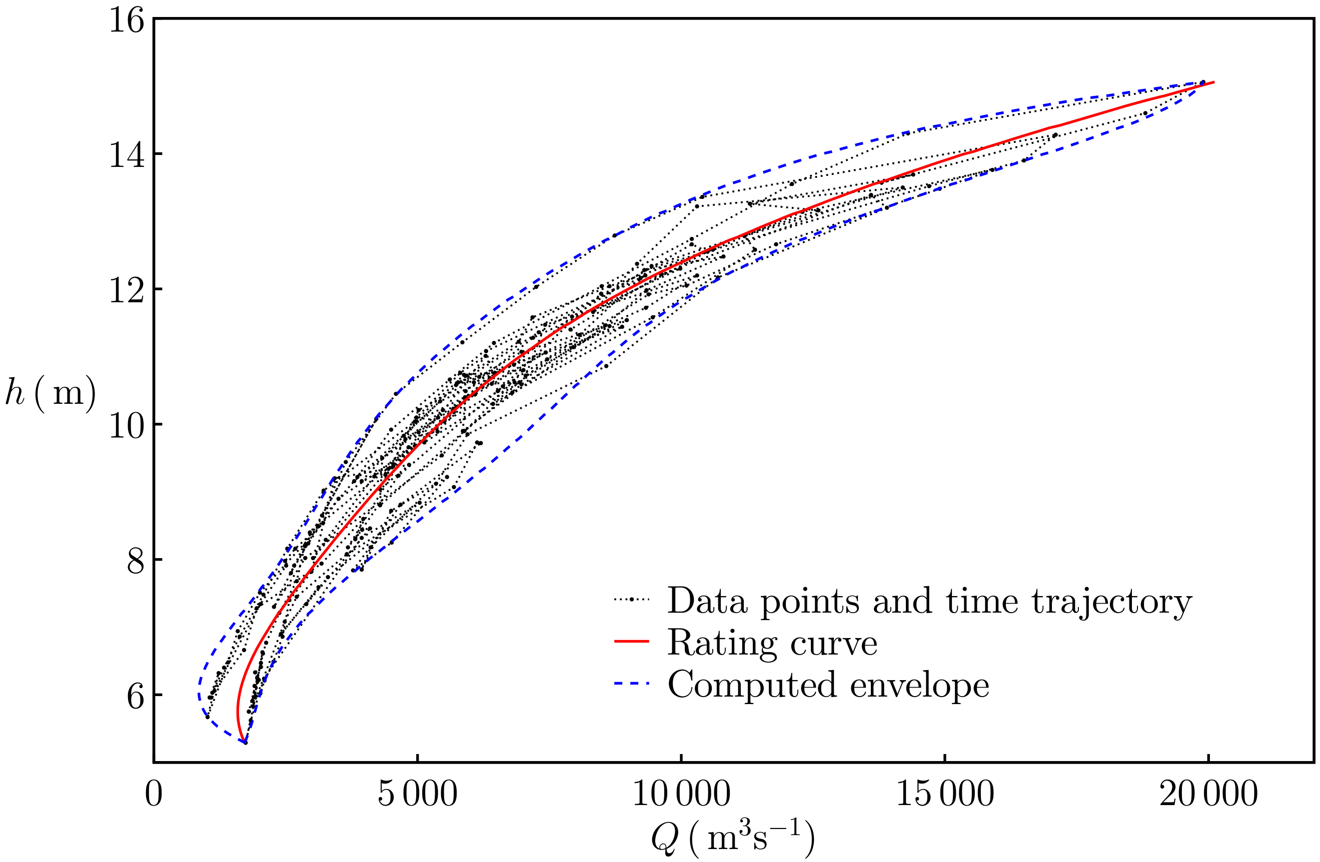
F: over the intervening time, flow has been small and almost constant, however the time has been enough to reduce the bed-forms and pack the bed grains to some extent. Now another flood starts to arrive, and this time, instead of following the flat-bed curve, it already starts from a finite resistance.

G: after this, the history of the stage will still depend on the history of the flow and the characteristics of the rate of change of the bed.

## The rating trajectory and rating envelope — generalisations of the single curve

- The ephemeral nature of the resistance means that in highly mobile bed streams it is not possible to calculate accurately the flow at any later time. It is not known what flows and bed changes will occur in the future up until the moment the curve is required to give a flow from a routine stage measurement.
- The view of the rating curve here is that it is an approximate curve passing through a more-or-less scattered cloud of points, where at least some of that scatter is due to fluctuations in the preceding flows and instantaneous state of the bed when each point was determined.
- If a long period of time is considered, with many flow events of different magnitudes, the flow trajectory will consist of a number of different paths and loops, the whole adding up to a complicated web occupying a limited region.
- Usually, however, one does not measure rating points very often, and instead of a continuous flow trajectory following an identifiable path, one just sees a discrete number of apparently-random points, occupying a more-or-less limited region.
- The more stable the bed, the less will be the scatter.
- Generally the points will fall in a band between a lower boundary, corresponding to smaller resistance, with an armoured bed, and an upper boundary corresponding to greater resistance with individual grains protruding and possibly bed-forms prominent when, for a given flow, the water will be deeper and stage higher.

- This leads to an extension of the idea of a single rating curve: that in a stream of variable bed conditions, one can never know the situation when rating data is actually to be used to predict a flow, and so it might be helpful also to compute a *rating envelope*, to provide expressions for curves approximating both upper and lower bounds.
- Suggested procedure:
  - Calculate the approximation to all the points, the rating curve
  - Then delete those points which lie *below* it.
  - Approximate the remaining points, and repeat as many times as necessary, to give the upper envelope.
  - Repeat, successively deleting all points *above* each curve.
  - As approximately half the data points are lost with each pass, the number of passes is limited. In practice what one would be doing is approximating the 1/8 or 1/16, say, of all data points, those which lie furthest from the approximation to all the points.




The figure shows an example for three years (1995–1997) of gaugings from Station 41 on the Red River, Viet Nam. The flow trajectory has several large loops, barely visible in the figure. Four passes of the halving procedure for each of the upper and lower envelopes were applied, starting with 217 data points, at the end there were about $217/2^4 \approx 15$ for each envelope. It can be seen that the method worked well.