

to substantially coincide with the a curve corresponding to exposed boulders on top of the bed occupying 0.2 of the surface area. Of course, with a number of these grains thus exposed, the resistance is greater. We assigned a value of  $\delta = 0.5$  to this intermediate state.

- Substituting  $\delta = 1$  in equation (3.18) gives the third curve on the figure, passing through what we believe is the third grouping of particles. This is probably the state for the maximum resistance for a stable bed corresponding to exposed grains occupying something like 50% of the surface area. Any more such grains will cause shielding of particles, the bed will start to resemble the co-planar case, and resistance will actually be reduced.
- Further evidence supporting our assertions is obtained from the expression proposed by Yen (2002, eqn 19), who considered results from a number of experimental studies using fixed impermeable beds. We used his formula, converted to  $\Lambda = \lambda/8$ , used an infinite Reynolds number, and converted his equivalent sand roughness  $\varepsilon_s = 2\varepsilon_{84}$ . It can be seen that the curve passes (left to right) from our curve  $\delta = 1$  for small particles, which are unlikely to have the tops levelled so that particles are exposed, to the second curve for larger particles, more likely to be levelled in the laboratory experiments, with  $\delta = 0$ . Yen obtained the approximation for  $\lambda$ :

$$\lambda = \frac{1}{4} \left( -\log_{10} \left( \frac{1}{12} \frac{k_s}{A/P} + \frac{1.95}{R^{0.9}} \right) \right)^{-2}, \quad (3.19)$$

where  $R$  is the channel Reynolds number  $R = (Q/P) / \nu$ , in which  $\nu$  is the kinematic viscosity.

- The logarithmic formula we obtained above, equation (3.6), leads to, if we use  $\varepsilon_s = 2\varepsilon_{84}$ , and

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as  $\gamma = \sqrt{8/\lambda} = 1/\sqrt{\Lambda}$ ,

$$\Lambda = \left( \frac{1}{\kappa} \ln \frac{11.}{\varepsilon} \right)^{-2} = (6.0 - 2.5 \ln (2\varepsilon_{84}))^{-2}, \quad (3.20)$$

giving results quite similar to those from Yen's formula.

- For points above the third curve almost all experimental points had shear stresses greater than the critical one necessary for movement. If particles move, not only do many particles protrude above others, increasing the stress, but there is the additional force required to maintain the sliding and rolling and jostling of all the particles. Hence, the resistance is greater. And, if there is a need to maintain particles in suspension, that will contribute also to resistance. We have shown the fourth curve as drawn for  $\delta = 2$ .

Hopefully the figure and approximating curves have given us an idea of the magnitudes and variation of the quantities, and maybe even some results for use in practice.