We have seen that A/P appeared naturally in the simple mechanical equilibrium calculation, and here we have found that it mimics the behaviour of $z_{\max}/h$, which is fortunate as it is a rather simpler quantity to calculate in practice, usually with no knowledge of the flow field. We will name A/P the *hydraulic mean depth*, and we will not use the conventional and misleading term "hydraulic radius" (dt. after Strickler – "Profil- oder hydraulischer Radius"). In channels that are wide, which is most, $P \approx B$ and A/P is about the same as the geometric mean depth A/B.

For rectangular channels that are not rectangular we have presented no results. Our suggestion is that A/P will still be a plausible approximation, and it already appears in the Chézy-Weisbach and Gauckler-Manning-Strickler equations (which, officially, we do not yet know in this course!). The use in those equations was justified by Keulegan (1938), however there is much wrong with that work, mathematically correctly integrating logarithmic velocity distributions over various shapes of cross-section but without any attention to real flows in channels.

Our suggested channel flow formula, replacing h by A/P in equation (3.2) is

$$U = \frac{Q}{A} = \frac{1}{\kappa}\sqrt{g\frac{A}{P}S}\left(\ln\frac{30/\mathrm{e}}{k_{\mathrm{s}}/\left(A/P\right)}\right), \qquad (3.4)$$

where $30/\mathrm{e} \approx 11.0$, which is often rounded to 12.

3.2 Theoretical/experimental flow formula

We introduce the symbol ε for the relative roughness

$$\varepsilon = \frac{k}{A/P} = \frac{\text{Grain size}}{\text{Hydraulic mean depth}} \approx \frac{\text{Grain size}}{\text{Depth}},$$

also for equivalent uniform sand grain size, $\varepsilon = k_{\mathrm{s}}/(A/P)$. Our flow formula (3.4) is then written in generalised form

$$U = \frac{Q}{A} = \gamma\sqrt{g\frac{A}{P}S}. \tag{3.5}$$

in which γ is a function of the relative roughness ε:

$$\gamma = \frac{1}{\kappa}\ln\frac{30/\mathrm{e}}{\varepsilon} \approx \frac{1}{\kappa}\ln\frac{11.}{\varepsilon}. \tag{3.6}$$

We now have a flow formula for steady uniform flow in a channel based on simple theory and experimental observations. This is an important result. We will soon see how the most common flow formula in practice is an approximation to this, but where this has the advantage that it is an explicit formula for the resistance coefficient in terms of the equivalent relative roughness of uniform grains.

Relative unimportance of grain size

In fact, γ, although all-important for us, is relatively slowly varying with grain size. Consider a small change in the relative roughness $\varepsilon\,(1+\Delta)$. The relative change in the factor γ is

$$\frac{\Delta\gamma}{\gamma} = \frac{\ln\left(11/(\varepsilon\,(1+\Delta))\right)}{\ln\left(11/\varepsilon\right)} - 1 \approx \frac{-\Delta}{\ln\left(11/\varepsilon\right)},$$

having expanded the logarithm as a power series $\ln\left(1+\Delta\right) = \Delta + \ldots$. Now for a value of $\varepsilon = 0.001$ (a $1\,\mathrm{mm}$ grain in $1\,\mathrm{m}$ of water), a relative change of $\Delta = 50\%$ gives a relative change in the factor γ in the equation of only -5%. Even for a much rougher case of $\varepsilon = 0.1$, the same relative change of 50% in grain size changes the left side by just -11%. It does not matter so much if we cannot specify the bed conditions all that accurately.

3.3 The Gauckler-Manning-Strickler formula

We now show that the G-M-S formula is an approximation to the expression we have obtained.

On Figure 3.6 is shown how the dimensionless factor γ varies as a function of relative roughness ε, given by equation (3.6) from experimental fluid mechanics. It is actually possible to approximate that curve closely using a monomial function a/ε^{μ}. The best values of a and μ can be found by performing a least-squares fit over 11 points equally-spaced in $\log\varepsilon$ between $\varepsilon = 0.001$ and 0.1. The result obtained was $\mu = 1/7.00$, which is a surprising coincidence. Now, setting $\mu = 1/7$ and determining just a by optimisation, a value of $a \approx 8.9$ was obtained:

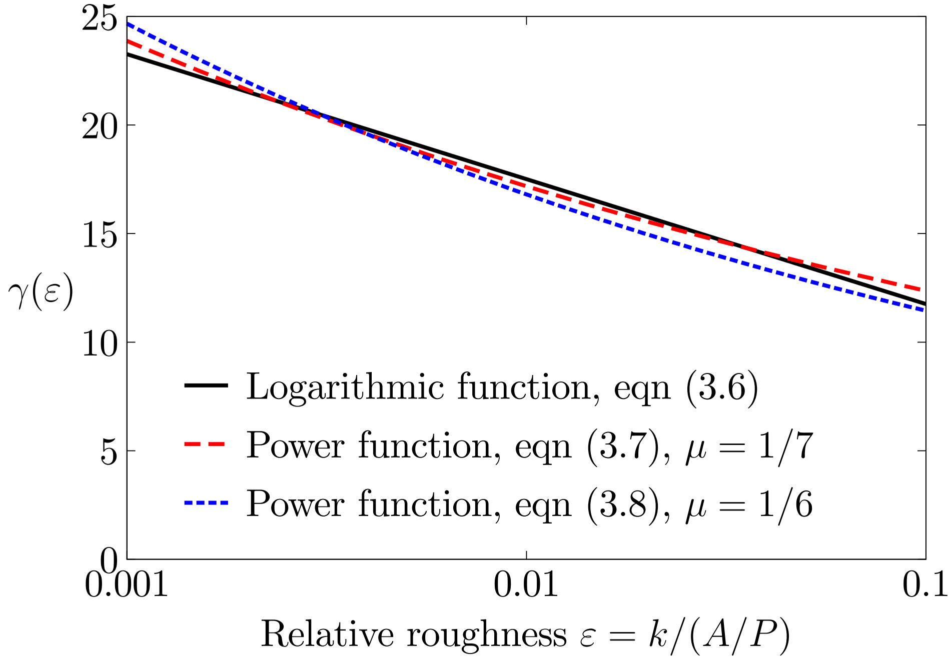

Figure 3.6:

$$\gamma = 8.9 \left(\frac{A/P}{k_{\mathrm{s}}}\right)^{1/7}, \tag{3.7}$$

with close agreement with the expression from the logarithmic velocity distribution shown in the figure. Using such a value would give us another flow formula, very similar to the

Gauckler-Manning-Strickler (G-M-S) formula. It is

$$U = \frac{Q}{A} = 8.9 \left(\frac{A/P}{k_\mathrm{s}}\right)^{1/7} \sqrt{g\frac{A}{P}S},$$

however unlike G-M-S this is explicit in terms of bed grain size. We do not want to proliferate an already crowded field of flow formulae, so we will not consider this further. It does, however, suggest the next step. The logarithmic function is now approximated again, this time by a function $b/\varepsilon^{1/6}$, where b is a constant. This constant can be determined by performing a similar least-squares fit, giving a value of $b \approx 7.8$ such that

$$\gamma = 7.8 \left(\frac{A/P}{k_\mathrm{s}}\right)^{1/6}, \tag{3.8}$$

with satisfactory results shown in Figure 3.6, showing that this is also quite a good approximation to the logarithmic function. Substituting into the flow formula, equation (3.5) and re-writing, we obtain

$$U = \frac{Q}{A} = \frac{7.8\sqrt{g}}{k_\mathrm{s}^{1/6}} \left(\frac{A}{P}\right)^{2/3} \sqrt{S} = k_\mathrm{St} \left(\frac{A}{P}\right)^{2/3} \sqrt{S} = \frac{1}{n} \left(\frac{A}{P}\right)^{2/3} \sqrt{S}, \tag{3.9}$$

which is simply the Gauckler-Manning-Strickler equation, where k_St is the Strickler coefficient and $n = 1/k_\mathrm{St}$ is the Manning coefficient! Unlike the G-M-S equation, this has given an explicit expression for the Strickler coefficient

$$k_\mathrm{St} = \frac{7.8\sqrt{g}}{k_\mathrm{s}^{1/6}}. \tag{3.10}$$