Institute of Hydraulic and Water Resources Engineering, Vienna University of Technology

River Engineering

Solution SheetDischarge Measurement and Rating Curves

1. *In lectures we had a scheme for obtaining the mean velocity in the vertical by measuring the velocity at two points. A simpler scheme is where the velocity is measured at a single point $0.6\,h$ from the surface. Show that the velocity at this point $0.4\,h$ (above the bottom) is a close approximation to the mean velocity for velocity profiles in (a) & (b) by integrating to find the mean velocity, and then finding the value of z for which the velocity is equal to the mean.*

   a. *The Prandtl-von Kármán law for turbulent flow over a rough bed:*

   $$u = \frac{u_*}{\kappa} \log \frac{z}{z_0}.$$

   You will need the result that $\int \log z\, dz = z \log z - z + C$. (Ans: $z/h = e^{-1} = 0.37$*, such that the relative depth of measurement should be* 0.63).

   Here it is simpler to use a local z co-ordinate based on the bed.

   $$\begin{aligned} \bar{u} &= \frac{1}{h}\int_0^h u\, dz = \frac{u_*}{\kappa h}\int_0^h (\log z - \log z_0)\; dz \\ &= \frac{u_*}{\kappa h}\left(z(\log z - 1) - z \log z_0\right)\Big|_0^h \\ &= \frac{u_*}{\kappa h}\left(h(\log h - 1) - h \log z_0\right) = \frac{u_*}{\kappa}\left(\log \frac{h}{z_0} - 1\right) \end{aligned}$$

   Now, let Z be the point at which the velocity is equal to this, such that

   $$\bar{u} = \frac{u_*}{\kappa} \log \frac{Z}{z_0}.$$

   Equating the two expressions we have

   $$\log \frac{h}{z_0} - 1 = \log \frac{Z}{z_0},$$

   with solution $\log h/Z = 1$, or $Z/h = e^{-1} \approx 0.37$.

   b. *The simple* $1/7$ *law, sometimes used as a simpler model for turbulent velocity distributions:*

   $$u = U \left(\frac{z}{h}\right)^{1/7},$$

   where U *is the surface velocity. (Ans:* $z/h = 0.39$*relative depth* 0.61*)*

   Similarly,

   $$\begin{aligned} \bar{u} &= \frac{1}{h}\int_0^h u\, dz = \frac{U}{h}\int_0^h \left(\frac{z}{h}\right)^{1/7}\; dz = U\int_0^1 \left(\frac{z}{h}\right)^{1/7}\; d\left(\frac{z}{h}\right) \\ &= U \times \frac{7}{8}, \end{aligned}$$

   and, when $u = \bar{u}$,

   $$\bar{u} = U \left(\frac{Z}{h}\right)^{1/7},$$

and equating the two expressions gives

$$\frac{Z}{h} = \left(\frac{7}{8}\right)^7 \approx 0.39.$$

c. *And, do it for the general power law*

$$u = U\left(\frac{z}{h}\right)^\nu .$$

(*Ans:* $z/h = (1+\nu)^{-1/\nu}$*. Plot it and be astonished how little it varies for* $0 < \nu < 0.25$*. Then take the limit as* $\nu \to 0$ *and be astonished that it approaches the value* $e^{-1} = 0.37$*, the same as for the logarithmic law. This is a glorious coming-together of mathematics, for it is Euler's formula for* e*:* $\lim_{x\to 0}(1+x)^{1/x}$ *!*)

Consider the plot – there is little variation, and in practical applications the difference between 0.37 and 0.41 for the setting of a velocity meter is negligible.

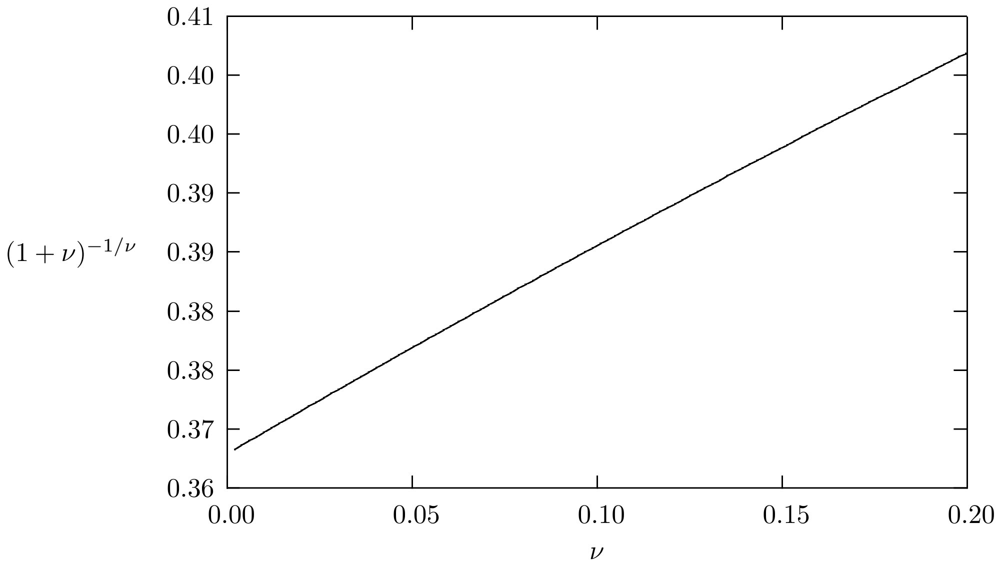

Figure 1. Variation of point where $u = \bar{u}$

2. *The Australian water industry uses a non-SI unit for flow, namely Megalitre per day (ML/d).*

a. *Verify that a cube* $10\,\mathrm{m} \times 10\,\mathrm{m} \times 10\,\mathrm{m}$ *contains 1 Megalitre.*

$$1000\,\mathrm{m}^3 = 1000 \times 1000\,\mathrm{L} = 10^6\,\mathrm{L} = 1\,\mathrm{ML}$$

b. *It is often said that 1 Megalitre is roughly the size of a* $50\,\mathrm{m}$ *Olympic swimming pool. Make some estimates of other dimensions and test the truth of that statement.*

Assume $50.000\,\mathrm{m}$ long (correct to the nearest mm – unusual for hydraulics!), say, what, $20\,\mathrm{m}$ wide, and $1.5\,\mathrm{m}$ deep, $50 \times 20 \times 1.5\,\mathrm{m}^3 = 1.5\,\mathrm{ML}$.

c. *Show that* $1\,\mathrm{m}^3\,\mathrm{s}^{-1} = 86.4$ *ML/d. Often, "*$\mathrm{m}^3\,\mathrm{s}^{-1}$*" is referred to as "cumec".*

$$1\,\mathrm{m}^3\,\mathrm{s}^{-1} = 24 \times 3600\,\mathrm{m}^3\mathrm{d}^{-1} = \frac{24 \times 3600}{1000}\mathrm{ML/d} = 86.4\,\mathrm{ML/d}$$

3. *Hydrographers sometimes use a unit of velocity of km/day for calibrating their propeller meters and presenting their data. This is not as silly as it sounds.*

a. *Verify that if velocities in km/day are integrated over cross sectional areas specified in m*^{2}*, the result is directly Ml/d. The velocity in km/day gives a practical idea of the distance that the* *water will travel in a day.*

$$\mathrm{km\,d^{-1}} \times \ \mathrm{m^2} = 1000\,\mathrm{m^3 d^{-1}} = 1\,\mathrm{ML/d}$$

b. *Verify that the velocity in km/day is also roughly the velocity in* $\mathrm{cm\,s^{-1}}$*, also useful for practical considerations, and show that a velocity of 30 km/day is* $34.7\,\mathrm{cm\,s^{-1}}$.

$$\begin{aligned} 1\,\mathrm{km\,d^{-1}} &= \frac{1 \times 1000 \times 100}{24 \times 3600}\,\mathrm{cm\,s^{-1}} = 1.16\,\mathrm{cm\,s^{-1}} \\ 30\,\mathrm{km\,d^{-1}} &= \frac{30 \times 1000 \times 100}{24 \times 3600}\,\mathrm{cm\,s^{-1}} = 34.7\,\mathrm{cm\,s^{-1}} \end{aligned}$$

4. See lecture notes
5. See lecture notes