

Worked solution - Tutorial Sheet 2

restart;read "C:/JF/Software/Maple/Start.mpl";Digits:=4:

▼ Question 2

$$Q[r] := \frac{k[St] A^{\frac{5}{3}} \sqrt{S}}{P(A)^{\frac{2}{3}}};$$

Evaluate $c = \frac{dQ}{dA} - \text{diff}(Q[r], A)$;

$$\frac{5}{3} \frac{k_{St} A^{2/3} \sqrt{S}}{P(A)^{2/3}} - \frac{2}{3} \frac{k_{St} A^{5/3} \sqrt{S} \left(\frac{d}{dA} P(A) \right)}{P(A)^{5/3}} \quad (1.1)$$

Express that in terms of mean fluid speed U : $\text{subs}(k[St]=U/(A^{(2/3)}/P(A)^{(2/3)} * \text{sqrt}(S)), \%)$;

$$\frac{5}{3} U - \frac{2}{3} \frac{U A \left(\frac{d}{dA} P(A) \right)}{P(A)} \quad (1.2)$$

$$\text{subs} \left(\frac{d}{dA} P(A) = \frac{\frac{\partial}{\partial h} P(A)}{\frac{\partial}{\partial h} A}, \% \right)$$

$$\frac{5}{3} U - \frac{2}{3} \frac{U A \left(\frac{\partial}{\partial h} P(A) \right)}{P(A) \left(\frac{\partial}{\partial h} A \right)} \quad (1.3)$$

$c := \text{subs}(\text{Diff}(A, h) = B, \%)$;

$$c := \frac{5}{3} U - \frac{2}{3} \frac{U A \left(\frac{\partial}{\partial h} P(A) \right)}{P(A) B} \quad (1.4)$$

▼ Question 3

$B := W + 2 * m * h; A := h * (W + m * h); P := W + 2 * \text{sqrt}(1 + m * m) * h$;

$$B := W + 2 m h$$

$$A := h (W + m h)$$

$$P := W + 2 \sqrt{1 + m^2} h \quad (2.1)$$

Rewriting the expression above: $c := \frac{5U}{3} - \frac{2UA \left(\frac{\partial}{\partial h} P \right)}{3PB}$

$$c := \frac{5}{3} U - \frac{4}{3} \frac{U h (W + m h) \sqrt{1 + m^2}}{(W + 2 \sqrt{1 + m^2} h) (W + 2 m h)} \quad (2.2)$$

$$W := 10 : m := 2 : S := 0.0001 : k[St] := \frac{1}{0.03} : h := 2.5 : g := 9.8 :$$

▼ (a)

From GMS equation above $Q[r]$; $U := Q[r]/A$;

$$18.28$$

$$U := 0.4875 \quad (2.1.1)$$

▼ (b) Speeds of propagation

Very long wave speed: c ;

$$0.6838 \quad (2.2.1)$$

Wide channel approximation: $5/3 * U$;

$$0.8125 \quad (2.2.2)$$

"Dynamic" wave speed: $\text{sqrt}(g * A/B)$;

$$4.287 \quad (2.2.3)$$

▼ (c) Time of travel over 10km

$10000/c$; (Seconds)

$$14620. \quad (2.3.1)$$

$\%/3600$; (Hours)

$$4.061 \quad (2.3.2)$$

▼ Questions 4 & 5

Answered on question sheet.