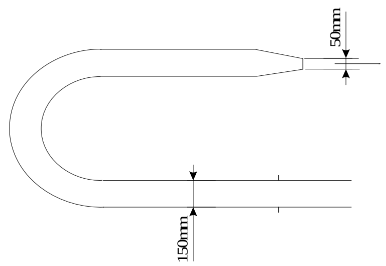
Hydraulics

Tutorial Sheet 6 – The momentum theorem

1. A jet of water issues from a nozzle at a speed of $6\,\mathrm{m\,s^{-1}}$ and strikes a stationary flat plate oriented perpendicular to the jet. The exit area of the nozzle is $645\,\mathrm{mm}^2$ and the Bousinesq momentum coefficient is $\beta = 1.05$. Draw a diagram of the flow plus a control volume and calculate the total force on the plate from the fluid in contact with it for two cases
   a. if the fluid travels parallel to the plate after impact, (Ans: $25.5\,\mathrm{N}$), and
   b. if each particle of fluid rebounds back in the direction from which it came without loss? (Ans: twice the previous value).
2. A flat plate is struck normally by a jet of water $50\,\mathrm{mm}$ in diameter with a velocity of $18\,\mathrm{m\,s^{-1}}$ and $\beta = 1.05$. After collision with the plate the water travels parallel to the plate and has no momentum in the direction perpendicular to the plate.Calculate
   a. the force on the plate when it is stationary, (Ans. $700\,\mathrm{N}$)
   b. the force when it moves in the same direction as the jet with a velocity of $6\,\mathrm{m\,s^{-1}}$. (Superimpose a uniform horizontal velocity on your system such that all motion is steady) (Ans. $311\,\mathrm{N}$)
3. The pipe bend and nozzle in the figure is bolted onto a pipe at 1, where it has a diameter of $150\,\mathrm{mm}$. It turns $180°$ in a horizontal plane and narrows down to $50\,\mathrm{mm}$ as it discharges the water into air. The discharge is 56.5 Ls^{-1}.



   a. Calculate mean velocity at 1 and at 2.
   b. Calculate the pressure at 1. Assume $\alpha = 1.3$ in the energy equation. If you are not familiar with the energy (Bernoulli) equation, use the result $530\,\mathrm{kPa}$.
   c. Calculate the net horizontal force exerted on the pipe bend and nozzle. Assume $\beta = 1.15$ in the momentum equation.
   d. Repeat (b) and (c) using the conventional approach with $\alpha = \beta = 1$.

   (Ans.: $3.19\,\mathrm{m\,s^{-1}}$and $28.8\,\mathrm{m\,s^{-1}}$; $530\,\mathrm{kPa}$; $11.5\,\mathrm{kN}$ to the left; $410\,\mathrm{kPa}$; $9.1\,\mathrm{kN}$ to the left – our use of Coriolis and Boussinesq coefficients is well-justified).
4. Water flows along a rectangular irrigation canal of width b and depth d, with discharge Q, Boussinesq momentum coefficient β, density ρ and gravitational acceleration g. You may assume that the pressure in the water is hydrostatic, as the streamlines are all parallel.
   a. Show that the magnitude of the momentum flux across any section is

$$\rho\left(\beta\frac{Q^2}{bd} + \tfrac{1}{2}gbd^2\right).$$

   (Note that the inertial momentum flux term can be obtained simply; while in this case the pressure contribution is non-trivial).
   b. Check that your equation is dimensionally homogeneous.
5. The force on the nozzle of a fire hose - a fire hose is $100\,\mathrm{mm}$ in diameter and is required to deliver

a stream of $20\,\mathrm{L\,s^{-1}}$. A nozzle is fixed to the hose which forces the jet of water to leave the hose with a diameter of $50\,\mathrm{mm}$. Use momentum principles to calculate the force on the nozzle. You may assume that the density of water is $1000\,\mathrm{kg\,m^{-3}}$, that the Boussinesq coefficient is $\beta = 1.2$ and the Coriolis coefficient $\alpha = 1.3$. (Ans: $315\,\mathrm{N}$)

6. A nozzle at a wheat shipping terminal is capable of delivering 0.7 bags per second into the hold of a ship. (One bag has a mass of $82\,\mathrm{kg}$ and a volume of 110L). The nozzle is circular in section with a diameter of $30\,\mathrm{cm}$, and is directed downwards and away from the wharf at an angle of $45°$. Draw a control volume for the stream of wheat and the hold of the ship. If six such nozzles are operating, estimate the force on the ship and what is its horizontal component tending to move the ship away from the wharf? (Ans: $375\,\mathrm{N}$, $265\,\mathrm{N}$).