Hydraulics

Tutorial Sheet 4 – Stability of floating bodies

1. A uniform wooden cylinder has a relative density of 0.6. Determine the minimum ratio of diameter to length so that it will float upright in water. (*Ans*: 1.386).

2. A square wooden beam of relative density σ has dimensions $L \times d \times d$, and floats in water such that the waterline cross section is a rectangle of dimensions $L \times d$.

   a. Show that the draught (depth to which the beam sinks) is σd.

   b. Show that the vertical distance from the centre of buoyancy to the metacentre is $d/12\sigma$.

   c. Sketch a cross-section and show that the vertical distance from the centre of buoyancy to the centre of gravity is $d/2 \times (1 - \sigma)$.

   d. Hence show that the stability or otherwise of the beam depends only on relative density in the form $1/12\sigma + (\sigma - 1)/2$.

   e. For what range of relative densities would the beam float without rotating to a new equilibrium position? (*Ans.*: $0 \leqslant \sigma \leqslant 0.211$, $0.789 \leqslant \sigma \leqslant 1$)

3. A raft is formed of three cylinders, each $1.2\,\mathrm{m}$ in diameter and $10\,\mathrm{m}$ long, placed parallel with their axes horizontal, the extreme breadth over the cylinders being $6\,\mathrm{m}$. When laden the raft floats with the cylinders half immersed and its centre of gravity $1.2\,\mathrm{m}$ above the centre cylinder axis. The waterline cross-section thus consists of three parallel rectangles of length $10\,\mathrm{m}$ with a distance of $2.4\,\mathrm{m}$ between centre-lines. Calculate the metacentric height. (You will need to use the fact that the centroid of a semicircle is $4/3\pi$ of the radius from the centre) (*Ans*: $6.95\,\mathrm{m}$).

   (The stability of catamarans and trimarans follows from the fact that their cross-section at the waterline is distributed as far from the centre-line of the boat as possible (to make I as large as possible), in the same way that structural members in bending try to ensure that the cross-section is far from the neutral axis.)

4. A rectangular pontoon $10\,\mathrm{m}$ by $4\,\mathrm{m}$ in plan, weighs $280\,\mathrm{kN}$ and floats in sea water of density $1025\,\mathrm{kg\,m^{-3}}$. A steel tube weighing $34\,\mathrm{kN}$ is placed longitudinally on the deck. When the tube is in a central position, the centre of gravity for the combined mass is on the vertical axis of symmetry $0.25\,\mathrm{m}$ above the water surface. Find

   a. the metacentric height, and

   b. the maximum distance the tube may be rolled laterally across the deck if the angle of heel is not to exceed 5°.

   (*Ans.*: $1.067\,\mathrm{m}$, $0.860\,\mathrm{m}$).