Hydraulics

Tutorial Sheet 2 – Fluid properties and pressures in fluids in static equilibrium

Unless advised otherwise, in all problems below the density of fresh water may be assumed to be $\rho = 1000\,\mathrm{kg\,m^{-3}}$, the density of seawater $1025\,\mathrm{kg\,m^{-3}}$, the acceleration due to gravity $9.8\,\mathrm{m\,s^{-2}}$, and the kinematic viscosity of water to be $\nu = 10^{-6}\,\mathrm{m^2\,s^{-1}}$.

1. Extracted from the Karlsruhe details on the German Weather Service:

Aktuell: 12.11.2006 - 23:00 Uhr Ortszeit		
Temperatur: 6.2°C	Luftdruck: 1021.0 hPa	Wind: 22 km/h / Süd-West

   Convert the air pressure ("Luftdruck") to an equivalent static head of (a) fresh water (in m), and (b) mercury of relative density 13.6 (in mm). (Ans. $1021 \times 10^2/1000/9.8 = 10.4\,\mathrm{m}$; $10.4/13.6 \times 1000 = 764\,\mathrm{mm}$)

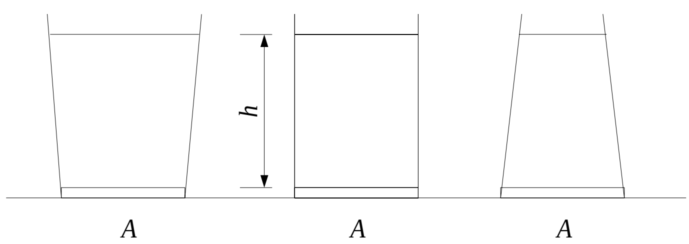
2. We need an engineering estimate of the order of magnitude of viscous effects in two flow situations:
   a. An hydraulic engineering situation: assume a river to be $3\,\mathrm{m}$ deep, a slope of 10^{-4}, to have a surface velocity of $1\,\mathrm{m\,s^{-1}}$ and of course a velocity on the bottom of 0. Consider a vertical column of water of base $1\,\mathrm{m} \times 1\,\mathrm{m}$, and
      i. estimate roughly the gravity force on the column and then the component parallel to the bed,
      ii. use Newton's law of viscosity, assuming a constant velocity gradient as a rough approximation, to calculate the force due to molecular viscosity on the bottom of the column and compare this with the gravitational force on the column.

      (*Ans.*: Component of gravitational force $\approx 3\,\mathrm{N}$, force due to molecular viscosity $\approx 3 \times 10^{-4}\,\mathrm{N}$, ratio of this to the gravitational force $\approx 10^{-4}$. Hence, *molecular* viscosity is unimportant. What we have not considered here, however, is the fact that in a turbulent flow like this, momentum transfer between slow and fast-moving fluid is by large parcels of fluid moving turbulently in the flow.)
   b. A mechanical engineering lubrication problem – consider the flow of oil of dynamic viscosity $7 \times 10^{-3}\,\mathrm{N\,s\,m^{-2}}$ in a machine bearing, where the flow increases uniformly from 0 to $10\,\mathrm{m\,s^{-1}}$ in $0.2\,\mathrm{mm}$. What is the shear stress on the bearing surface? How does this compare with (a) above? (*Ans.*: $350\,\mathrm{N\,m^{-2}}$; it is very much greater in this situation where the fluid is viscous and the velocity gradient is much larger than in (a).)

3. A mass of 50 kg sits on a piston of area 100 cm^2, which rests on water in a vertical cylinder. What is the pressure on the underside of the piston? What is the pressure in the water 1 m underneath the piston? (*Ans.* $49\,\mathrm{kPa}$, $59\,\mathrm{kPa}$).

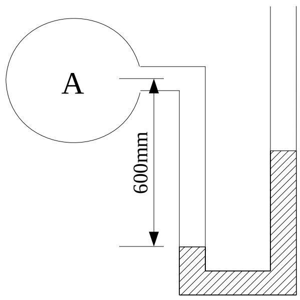
4. A diver is working in the sea $18\,\mathrm{m}$ below the surface. What is the total pressure there? Express this in terms of atmospheric pressure. (The pressure of one atmosphere $\approx 10^5\,\mathrm{Pa}$). (*Ans.* $181\,\mathrm{kPa} + 100 \pm 2\,\mathrm{kPa} \approx 280\,\mathrm{kPa} = 2.8\,\mathrm{bar}$).

5. Consider the three vessels of the same base area A, filled to the same depth of water h.What is the pressure of the fluid on the bottom of the container in each case? Why does this not always equal the pressure between the vessel and the supporting surface?



6. Calculate the equilibrium height difference Δh between the two pistons of mass m_1 and m_2 and area A_1 and A_2, which are free to move without friction in their respective cylinders, each filled with and connected by a fluid of density ρ. Check your answer for dimensional correctness. (*Ans.* $(m_1/A_1 - m_2/A_2)/\rho$).

7. In the figure, pipe A contains water, and the manometer fluid has a relative density of 2.94. The interface in the right tube is $400\,\mathrm{mm}$ higher than the interface in the left tube. What is the gauge pressure at A? (*Ans.*: $5650\,\mathrm{Pa}$).



8. Two pipes A and B contain water, as shown in the figure. A mercury (relative density 13.6) manometer is used to measure the pressure difference, by measuring the difference in meniscus levels. With the data as given in the figure, calculate the pressure difference. (*Ans.*: $41\,\mathrm{kPa}$).