Hydraulics

Solution Sheet 4 – Stability of floating bodies

1. *A uniform wooden cylinder has a relative density of 0.6. Determine the minimum ratio of diameter to length so that it will float upright in water. (Ans:* 1.386*).*

   Let D be diameter, L length, density of water ρ, relative density of cylinder σ. Let x be the draught.

$$\begin{aligned}\text{Weight of cylinder} &= \sigma\rho g\frac{\pi}{4}D^2L,\\ \text{Weight of displaced fluid} &= \rho g\frac{\pi}{4}D^2x.\end{aligned}$$

   Equating the two gives $x=\sigma L$.

   Now, to calculate BM we need the second moment of area about an axis through the centre of the circular waterline cross-section and the displaced volume:

$$\begin{aligned}I_G &= \frac{\pi D^4}{64} \quad\text{and}\quad V=\frac{\pi}{4}D^2x=\sigma\frac{\pi}{4}D^2L,\\ \therefore\quad \text{BM} &= \frac{I_G}{V}=\frac{\pi D^4/64}{\sigma\frac{\pi}{4}D^2L}=\frac{1}{16}\frac{D^2}{\sigma L}.\end{aligned}$$

   From a simple side elevation we can conclude that BG$=L/2\,(1-\sigma)$, so that

$$\text{GM}=\text{BM}-\text{BG}=\frac{1}{16}\frac{D^2}{\sigma L}-\frac{L}{2}\left(1-\sigma\right).$$

   For limiting stability we require GM $\geqslant 0$, which on the limit GM$=0$ gives a quadratic equation

$$\frac{D}{L}=2\sqrt{2\sigma\left(1-\sigma\right)}=2\sqrt{2\times 0.6\left(1-0.6\right)}=1.386.$$

   It seems that for larger D/L the block will be more stable, so this is the minimum.

2. *A square wooden beam of relative density σ has dimensions $L\times d\times d$, and floats in water such that the waterline cross section is a rectangle of dimensions $L\times d$.*

   a. *Show that the draught (depth to which the beam sinks) is σd.*

   b. *Show that the vertical distance from the centre of buoyancy to the metacentre is $d/12\sigma$.*

   c. *Sketch a cross-section and show that the vertical distance from the centre of buoyancy to the centre of gravity is $d/2\times(1-\sigma)$.*

   d. *Hence show that the stability or otherwise of the beam depends only on relative density in the form $1/12\sigma+(\sigma-1)/2$.*

   e. *For what range of relative densities would the beam float without rotating to a new equilibrium position? (Ans.: $0\leqslant\sigma\leqslant 0.211$, $0.789\leqslant\sigma\leqslant 1$)*

   The solution proceeds very closely to above. By an identical procedure we obtain the draught $x=\sigma d$.

   Now, to calculate BM we need the second moment of area about an axis through the centre of the circular waterline cross-section and the displaced volume:

$$\begin{aligned}I_G &= \frac{1}{12}Ld^3 \quad\text{and}\quad V=\sigma Ld^2,\\ \therefore\quad \text{BM} &= \frac{I_G}{V}=\frac{Ld^3/12}{\sigma Ld^2}=\frac{d}{12\sigma}.\end{aligned}$$

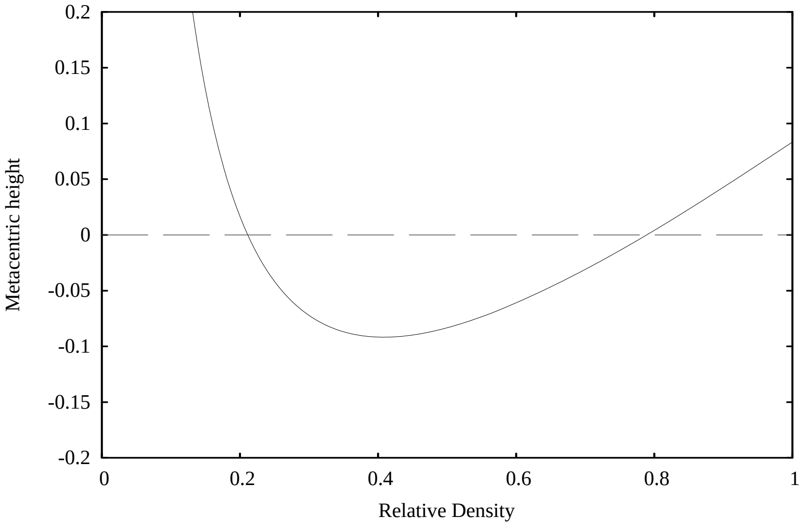
From a simple side elevation we can conclude that BG$= d/2\,(1-\sigma)$, so that

$$\begin{aligned}\text{GM} &= \text{BM} - \text{BG} = \frac{d}{12\sigma} - \frac{d}{2}\,(1-\sigma)\,. \\ \therefore \quad \frac{\text{GM}}{d} &= \frac{1}{12\sigma} - \frac{1}{2}\,(1-\sigma)\,.\end{aligned}$$

For limiting stability we require GM $\geqslant 0$, which on the limit GM$=0$ gives a quadratic equation with solutions

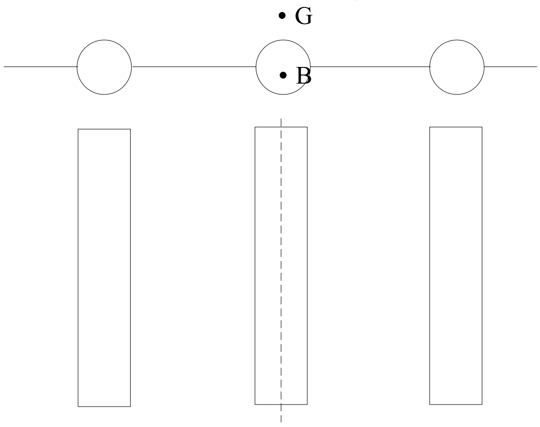
$$\sigma = \frac{1}{2} \pm \frac{\sqrt{3}}{6} = 0.211,\ 0.789.$$

Now we plot the quantity GM$/d$ for a range of σ:



and it is clear that for stability $0 \leqslant \sigma \leqslant 0.211$, $0.789 \leqslant \sigma \leqslant 1$.

3. *A raft is formed of three cylinders, each* $1.2\,\text{m}$ *in diameter and* $10\,\text{m}$ *long, placed parallel with their axes horizontal, the extreme breadth over the cylinders being* $6\,\text{m}$*. When laden the raft floats with the cylinders half immersed and its centre of gravity* $1.2\,\text{m}$ *above the centre cylinder axis. (The waterline cross-section thus consists of three parallel rectangles of length* $10\,\text{m}$ *with a distance of* $2.4\,\text{m}$ *between centre-lines.) Calculate the metacentric height. (Ans:* $6.95\,\text{m}$*).*



Each cylinder is $1.2\,\text{m}$ wide, and as the overall breadth is $6\,\text{m}$, the distance y between the centrelines of the cylinders is $2y + 1.2 = 6$, thus $y = 2.4\,\text{m}$.

The second moment of area about the centreline of each cylinder is $bd^3/12$, hence

$$
\begin{aligned}
I &= \frac{bd^3}{12} + 2\times\left(\underbrace{\frac{bd^3}{12} + bd\times y^2}_{\text{Parallel axis theorem}}\right) = \frac{bd^3}{4} + 2bdy^2 = \\
&\quad \frac{10\times 1.2^3}{4} + 2\times 10\times 1.2\times 2.4^2 = 142.6\,\mathrm{m}^4. \\
V &= 3\times\tfrac{1}{2}\times\frac{\pi\times 1.2^2}{4}\times 10 = 16.97\,\mathrm{m}^3 \\
\mathrm{BM} &= \frac{I}{V} = \frac{142.6}{16.97} = 8.40\,\mathrm{m}
\end{aligned}
$$

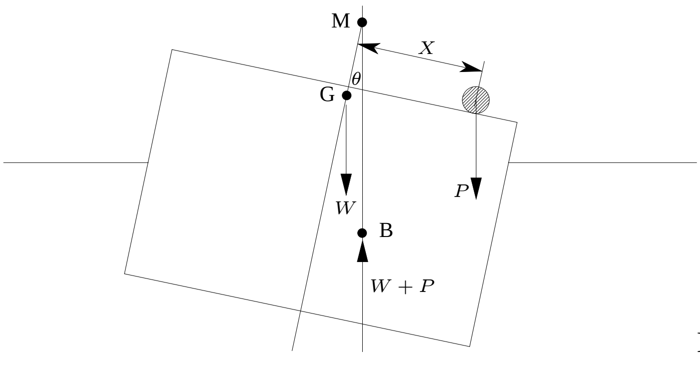
The centre of buoyancy of the three semicircles is
$4/(3\pi)\times 1.2/2 = 0.255$ below the waterline,
hence $\mathrm{BG} = 1.2 + 0.255 = 1.455\,\mathrm{m}$, and $\mathrm{GM} = \mathrm{BM} - \mathrm{BG} = 6.95\,\mathrm{m}$.

4. *A rectangular pontoon* $10\,\mathrm{m}$ *by* $4\,\mathrm{m}$ *in plan, weighs* $280\,\mathrm{kN}$ *and floats in sea water of density* $1025\,\mathrm{kg\,m^{-3}}$. *A steel tube weighing* $34\,\mathrm{kN}$ *is placed longitudinally on the deck. When the tube is in a central position, the centre of gravity for the combined mass is on the vertical axis of symmetry* $0.25\,\mathrm{m}$ *above the water surface. Find*
   a. *the metacentric height, and*
   b. *the maximum distance the tube may be rolled laterally across the deck if the angle of heel is not to exceed* $5°$.

$$
\begin{aligned}
\text{Weight of pontoon+load} &= 280 + 34 = 314\,\mathrm{kN} \\
\text{Weight of seawater displaced} &= 1025\times 9.8\times 10\times 4\times \text{Draught} \\
\therefore \text{Draught} &= \frac{314\times 1000}{1025\times 9.8\times 10\times 4} = 0.781\,\mathrm{m} \\
\mathrm{BM} &= \frac{I}{V} = \frac{\frac{1}{12}\times 10\times 4^3}{4\times 10\times 0.781} = 1.707\,\mathrm{m}
\end{aligned}
$$

The centre of gravity is $0.25 + \text{Draught}/2$ above B, $\therefore \mathrm{BG} = 0.25 + 0.781/2 = 0.640\,\mathrm{m}$

$$\mathrm{GM} = \text{BM-BG} = 1.707 - 0.640 = 1.067\,\mathrm{m}$$



Now consider the figure shown, where

W is the weight of the ship. Taking moments about G and for small θ such that $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$,

$$
\begin{aligned}
-P\times X + (W+P)\times \mathrm{GM}\times\theta = 0, \therefore X &= \frac{(W+P)\times \mathrm{GM}\times\theta}{P} \\
&= \frac{314\times 1.067\times 5\pi/180}{34} \\
&= 0.860\,\mathrm{m}
\end{aligned}
$$