Hydraulics

Solution Sheet 2 – Fluid properties and pressures in fluids in static equilibrium

1. *Convert the air pressure ("Luftdruck") to an equivalent static head of (a) fresh water (in m), and (b) mercury of relative density 13.6 (in mm).*

   (a) $p = \rho g h$, $h = p/\rho g = 1021 \times 10^2/1000/9.8 = 10.4\,\mathrm{m}$

   (b) $h = 10.4/13.6 = 0.765\,\mathrm{m} = 765\,\mathrm{mm}$.

2. *We need an engineering estimate of the order of magnitude of viscous effects in two flow situations:*

   a. *An hydraulic engineering situation: assume a river to be* $3\,\mathrm{m}$ *deep, a slope of* 10^{-4}*, to have a surface velocity of* $1\,\mathrm{m\,s^{-1}}$ *and of course a velocity on the bottom of* 0*. Consider a vertical column of water of base* $1\,\mathrm{m} \times 1\,\mathrm{m}$*, and*

      i. *estimate roughly the gravity force on the column and then the component parallel to the bed,*

      ii. *use Newton's law of viscosity, assuming a constant velocity gradient as a rough approximation, to calculate the force due to molecular viscosity on the bottom of the column and compare this with the gravitational force on the column.*

   This is meant to give an idea of the lack of importance of molecular viscosity in environmental engineering problems.

   Vertical gravity force on a column of $1\,\mathrm{m} \times 1\,\mathrm{m} \times 3\,\mathrm{m} = \rho g V = 1000 \times 9.8 \times 1 \times 1 \times 3 = 29400\,\mathrm{N}$.

   Component parallel to bed – we multiply by the sine of the slope angle, which here we can take to be the slope, $2.94 \approx 3\,\mathrm{N}$.

   Velocity gradient $du/dz \approx (1-0)/3 = 1/3\,\mathrm{s^{-1}}$ (a very rough approximation of course),

$$\begin{aligned}\text{Shear stress on bottom} &= \mu\frac{du}{dz} = \rho\nu\frac{du}{dz}\\ &= 1000 \times 10^{-6} \times 1/3\\ &\approx 3 \times 10^{-4}\,\mathrm{N\,m^{-2}}\end{aligned}$$

   Hence the force on the bottom is $1 \times 1 \times 3 \times 10^{-4}\,\mathrm{N}$.

$$\text{Ratio of Viscosity/Gravity} \approx \frac{1 \times 1 \times 3 \times 10^{-4}}{2.94} \approx 1 \times 10^{-4}$$

   Hence, *molecular* viscosity is unimportant. What we have not considered here, however, is the fact the in a turbulent flow like this, momentum transfer between slow and fast-moving fluid is by large parcels of fluid moving turbulently in the flow.

   b. *A mechanical engineering lubrication problem – consider the flow of oil of dynamic viscosity* $7 \times 10^{-3}\,\mathrm{N\,s\,m^{-2}}$ *in a machine bearing, where the flow increases uniformly from 0 to* $10\,\mathrm{m\,s^{-1}}$ *in* $0.2\,\mathrm{mm}$*. What is the shear stress on the bearing surface? How does this compare with (a) above? (Ans.:* $350\,\mathrm{N\,m^{-2}}$*; it is very much greater in this situation where the fluid is viscous and the velocity gradient is much larger than in (a).)*

   Machine bearing

$$\begin{aligned}\frac{du}{dz} &= \frac{10-0}{0.2 \times 10^{-3}} = 50000\\ \tau &= \mu\frac{du}{dz} = 7 \times 10^{-3} \times 50000 = 350\,\mathrm{N\,m^{-2}}.\end{aligned}$$

   It is very much greater in this situation where the fluid is viscous and the velocity gradient is much larger than in (a).

3. *A mass of 50 kg sits on a piston of area 100* cm^2*, which rests on water in a vertical cylinder. What is the pressure on the underside of the piston? What is the pressure in the water 1 m underneath the*

piston?

The mass is $50\,\mathrm{kg}$, therefore the weight force is $50 \times 9.8 = 490\,\mathrm{N}$, and the pressure is

$$\text{Pressure} = \frac{490}{100 \times 10^{-4}} = 49000\,\mathrm{Pa} = 49\,\mathrm{kPa}.$$

The additional pressure due to the lower elevation is $\rho g h = 1000 \times 9.8 \times 1 = 9800.0\,\mathrm{Pa} = 10\,\mathrm{kPa}$, hence the total pressure is $59\,\mathrm{kPa}$.

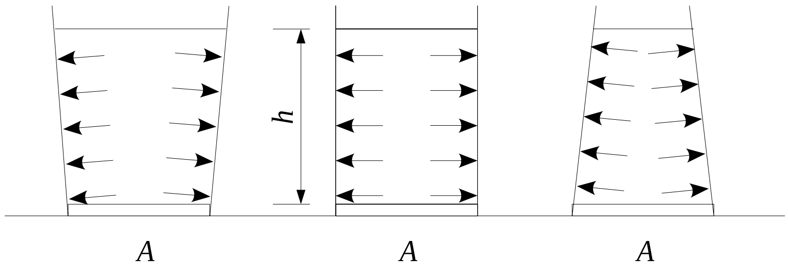
4. *A diver is working in the sea* $18\,\mathrm{m}$ *below the surface. What is the total pressure there? Express this in terms of atmospheric pressure. (The pressure of one atmosphere* $\approx 10^5\,\mathrm{Pa}$*).*

   $p = \rho g h = 10\mathbf{25} \times 9.8 \times 18 = 1.81 \times 10^5\,\mathrm{Pa} = 1.81\,\mathrm{bar}$.

   If we were to include atmospheric pressure, $181\,\mathrm{kPa} + 100 \pm 2\,\mathrm{kPa} \approx 280\,\mathrm{kPa} = 2.8\,\mathrm{bar}$.

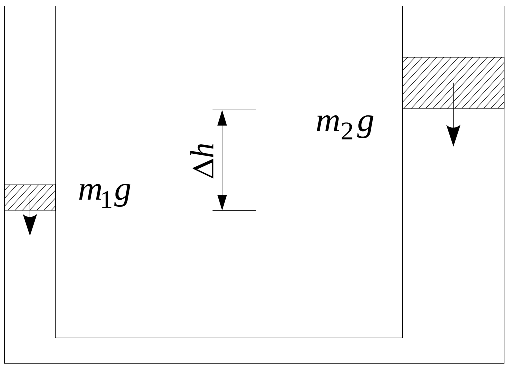
5. *Consider the three vessels of the same base area* A*, filled to the same depth of water* h*. What is the pressure of the fluid on the bottom of the container in each case? Why does this not always equal the pressure between the vessel and the supporting surface?*

   The pressure on the bottom (wet side) is $\rho g h$ in all three cases, hence the force of the water on the bottom is $\rho g h A$ in all three cases. In the middle case (neglecting weight of glass) this will be the force of glass on the desk, $\rho g V$, the weight of fluid, $= rghA$. In the first case the total volume is greater, $V > hA$, and the total force on the desk is greater. Considering the fluid forces on the inside, the force of the pressure on the sides is partly downwards, and hence part of the force is taken by the sides. The fluid force on the base in each case is the same, but the *total* vertical component of force of fluid on the vessel in each case is different, as is the total force on the support.



6. *Calculate the equilibrium height difference* Δh *between the two pistons of mass* m_1 *and* m_2 *and area* A_1 *and* A_2*, which are free to move without friction in their respective cylinders, each filled with and connected by a fluid of density* ρ*. Check your answer for dimensional correctness.*

   Consider the arrangement in the figure:



   Consider the pressure at the underside of piston 1:

$$p_1 = \frac{m_1 g}{A_1}.$$

The pressure on the underside of piston 2:

$$p_2 = \frac{m_2 g}{A_2}.$$

We apply the hydrostatic pressure equation $p + \rho g z = \text{constant}$ between these two points, and choose the origin in z to be at the underside of 1:

$$\begin{aligned} p_1 + \rho g z_1 &= p_2 + \rho g z_2 \\ \frac{m_1 g}{A_1} + 0 &= \frac{m_2 g}{A_2} + \rho g \Delta h, \end{aligned}$$

hence the solution

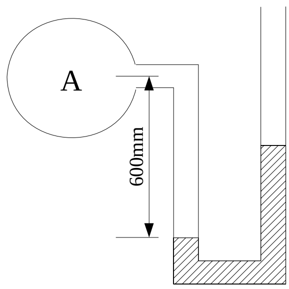
$$\Delta h = \frac{m_1}{\rho A_1} - \frac{m_2}{\rho A_2}.$$

Note that depending on the relative ratio of the m/A, the equilibrium position might be positive or negative.

Is this solution dimensionally homogeneous? The left has dimensions L, the right dimensions

$$\frac{\text{M}}{\text{ML}^{-3}\text{L}^2} = \text{L}.$$

7. *In the figure, pipe A contains water, and the manometer fluid has a relative density of* 2.94*. The interface in the right tube is* $400\,\text{mm}$ *higher than the interface in the left tube. What is the gauge pressure at A?*



(a)

Let O be the interface on the left side of the manometer. Applying the hydrostatic pressure equation between A and O:

$$\begin{aligned} p_\text{A} + \rho g z_\text{A} &= p_\text{O} + \rho g z_\text{O} \\ \therefore \quad p_\text{A} &= p_\text{O} + \rho g (z_\text{O} - z_\text{A}) \end{aligned}$$

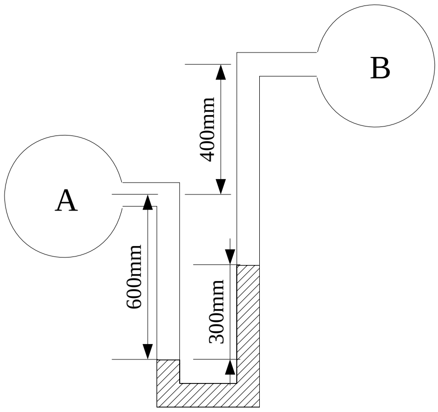
And now between O and C, the top of the manometer fluid open to the air (NOTE: the density of the fluid is now $\rho\sigma$, where σ is the relative density of the manometer fluid)

$$\begin{aligned} p_\text{O} + \rho\sigma g z_\text{O} &= p_\text{C} + \rho\sigma g z_\text{C} \\ \therefore \quad p_\text{O} &= p_\text{C} + \rho\sigma g (z_\text{C} - z_\text{O}) \end{aligned}$$

Substituting into the equation for p_A:

$$\begin{aligned} p_\text{A} &= p_\text{C} + \rho\sigma g (z_\text{C} - z_\text{O}) + \rho g (z_\text{O} - z_\text{A}) \\ \therefore p_\text{A} - p_\text{C} &= \rho g (\sigma (z_\text{C} - z_\text{O}) + z_\text{O} - z_\text{A}) \\ &= 1000 \times 9.8 \times (2.94\,(0.4) - 0.6) \\ &\approx 5.6\,\text{kPa} \end{aligned}$$

8. *Two pipes A and B contain water, as shown in the figure. A mercury (relative density 13.6) manometer is used to measure the pressure difference, by measuring the difference in meniscus levels. With the data as given in the figure, calculate the pressure difference.*



Let O be the interface on the left side of the manometer. Applying the hydrostatic pressure equation between A and O as before:

$$p_{\mathrm{A}} = p_{\mathrm{O}} + \rho g \left(z_{\mathrm{O}} - z_{\mathrm{A}}\right)$$

And now between O and C, the top of the manometer fluid, as before;

$$p_{\mathrm{O}} = p_{\mathrm{C}} + \rho \sigma g \left(z_{\mathrm{C}} - z_{\mathrm{O}}\right)$$

Between C and B:

$$p_{\mathrm{C}} = p_{\mathrm{B}} + \rho g \left(z_{\mathrm{B}} - z_{\mathrm{C}}\right),$$

and successively eliminating p_{C} and p_{O}:

$$\begin{aligned} p_{\mathrm{A}} &= p_{\mathrm{B}} + \rho g \left(z_{\mathrm{B}} - z_{\mathrm{C}}\right) + \rho \sigma g \left(z_{\mathrm{C}} - z_{\mathrm{O}}\right) + \rho g \left(z_{\mathrm{O}} - z_{\mathrm{A}}\right) \\ p_{\mathrm{A}} - p_{\mathrm{B}} &= \rho g \left(z_{\mathrm{B}} - z_{\mathrm{C}} + \sigma \left(z_{\mathrm{C}} - z_{\mathrm{O}}\right) + z_{\mathrm{O}} - z_{\mathrm{A}}\right) \\ &= 1000 \times 9.8 \times \left(0.7 + 13.6 \times (0.3) - 0.6\right) \\ &= 41\,\mathrm{kPa} \end{aligned}$$