

**Material derivatives**

1. By considering the temperature  $T$  of a particle of fluid at position  $\mathbf{r}$  and time  $t$  and at time  $t + \delta t$  at position  $\mathbf{r} + \delta \mathbf{r}$ , show that the apparent time rate of change of temperature of the particle is

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T.$$

2. Water flows steadily along a horizontal pipe which reduces uniformly in section from 20 cm diameter to 10 cm diameter in a distance of 1 m. The volume flow rate is  $0.02 \text{ m}^3 \text{ s}^{-1}$ . Obtain an expression for pipe diameter as a function of distance  $x$  from the beginning of the reducer, and obtain successively expressions for area, mean velocity and mean velocity gradient as functions of  $x$ . Hence or otherwise, calculate the acceleration of a fluid particle at the beginning of the reducer and at the end. (*Ans.*:  $0.41 \text{ m s}^{-2}$ ,  $13 \text{ m s}^{-2}$ ).